# Picking Up the Pieces: New Directions for Federal Anti-Gerrymandering Law After Rucho 

Alexander Karapetyan and Stephen M. Maurer


#### Abstract

For most of US history, the federal government let states conduct congressional elections with only minimal interference. This changed with Congress's passage of the Voting Rights Act (1964) and the Supreme Court's one-man-one-vote ("OMOV") decision in Carr (1965). Yet even then the Supreme Court stopped short of overruling gerrymandered congressional districts. Fifteen years ago, Justice Kennedy acknowledged that workable approaches still did not exist, but challenged litigants to do better. The Supreme Court's devastating opinion in Rucho v. Common Cause (2019) marks the end of this experiment.


The question remains whether some fresh departure can satisfy the Court's objections. This paper investigates the leading candidates. The most conservative possibility is to tighten traditional visual criteria like "contiguity" and "compactness" to make them binding. We present detailed numerical arguments showing that such rules would have to be so stringent as to deprive legislators of practically all discretion in drawing lines. This, however, would split communities at random. We argue that this (a) violates OMOV just as a deliberate gerrymander would, and (b) disrupts grassroots networks that voters rely on to educate themselves, making votes less valuable to those who cast them. It follows that OMOV is always improved by replacing random districting with determinate rules that track communities.

Our second candidate implements this strategy. Remarkably, recent social science research shows (a) that education and social pressure across voters play an important role in shaping community opinion, and (b) that the probability of such interactions between voters can be reliably estimated from an inverse square law. We describe a new open source software program that uses these insights to track community and test it against county-level population data from Texas. We show that the resulting maps are visually similar to those produced by legislatures; are fully determinate; efficiently balance OMOV constraints against respect for community; and are robust against manipulation. The chief downside is that some districts are discontinuous, although this is rare and could be mitigated even further. In the meantime, our algorithm provides a valuable safe harbor for States hoping to avoid future court challenges, an attractive model for reform legislation, and a transparent benchmark for exposing gerrymanders to public scrutiny.

# Picking Up the Pieces: Options for Federal Anti-Gerrymandering Law After Rucho 

Alexander Karapetyan ${ }^{1}$ and Stephen M. Maurer ${ }^{2}$

## I. Introduction

For most of US history, Congress and the Courts let states conduct federal elections with only minimal supervision. This changed with Congress's passage of the Voting Rights Act (1964) and the Supreme Court's one-man-one-vote ("OMOV") decision in Carr (1965). Yet even then the Court stopped short of overruling gerrymandered districts. Fifteen years ago, Justice Kennedy called on litigants to develop workable methods to fill this gap. This led to a multi-million dollar consulting industry and massive efforts to reduce traditional visual criteria like contiguity and compactness to mathematically rigorous rules. The majority's devastating critique in Rucho $v$. Common Cause marks the end of this experiment.

The question remains whether new methods can overcome the majority's objections. Probably the most obvious candidate is to make traditional geometric criteria like contiguity and compactness more determinate so that they become binding on legislatures. However, we show that this cannot be done without splitting large numbers of communities at random. This would violate OMOV in two ways. On the one hand, split communities dilute citizens' votes compared to communities that are not split. On the other hand, splitting also impairs voters' ability to educate each other and develop consensus views. Both effects make the franchise less valuable to those who exercise it. It follows that OMOV is always improved by replacing random districting with determinate rules that track communities.

Our second candidate implements this strategy. We argue that respect for community, or in modern language social networks, is a constitutional principle. Remarkably, recent social science research shows that ties between voters are reliably approximated by an inverse square law. We have written a new open source computer program ${ }^{3}$ that implements this principle using county-level population data from Texas. The results show that our algorithm generates reasonable (though occasionally discontinuous) boundaries; that it is robust against manipulation; and that it provides a reliable benchmark for detecting real world gerrymanders.

[^0]We proceed as follows. Part II reviews the long history of federal gerrymander law and the Rucho Court's definitive critique of existing methods. Part III asks whether traditional visual standards can be tightened to block most gerrymanders at an acceptable price. We present detailed numeric examples showing that the cost is almost certainly prohibitive. Part IV argues that the Framers identified democracy with preexisting geographic communities, and shows how the modern understanding of social networks updates the concept. This implies that congressional districts should be drawn to coincide with voter networks as much as possible. Part V describes novel open source software that implements this principle and evaluates it using county-level data from Texas. We show that the results are similar to maps normally produced by legislatures, are robust against small changes in how data are presented, and provide a reliable benchmark for identifying partisanship in existing maps. Part VI considers the limits of our approach, focusing on instances where alternative methods like at-large districts and even Web-voting might be appropriate. Part VII asks how our algorithm can be used to implement real world reforms. Part VIII presents a brief conclusion.

## II. Federal Redistricting Law: The Story So Far

We begin by reminding readers how gerrymanders are engineered and of the long history of efforts to rein them in. We conclude by describing the Rucho majority's rejection of current methods and the detailed objections that any future proposals must overcome.

## A. The Sin of Gerrymander

Federal gerrymander law is now buried under an insiders' vocabulary of terms like "packing," "cracking," and "wasted votes." ${ }^{4}$ Here we find it simpler and more illuminating to start with an example. Imagine three congressional districts $A, B$, and $C$, each of which is initially home to equal numbers of Republicans and Democrats. Then a partisan Democrat can redraw the borders so that District A gains Republicans on net from $B$ and C. For large enough transfers, the voters who remain in Districts B and C will then reliably elect Democrats. The price for this will be Republican landslides in District A, but under majority rule the extra margin counts for nothing. In the meantime, Democrats have gained one more seat than Republicans without changing a single voter's opinion. ${ }^{5}$

[^1]But if that is the sin, who has been sinned against? Not, the Supreme Court tells us, the two political parties. ${ }^{6}$ Rather, the focus is on individual citizens. More specifically, Republicans have seen their votes "wasted" through transfers to a district where their decision to vote cannot possibly change outcomes. Crucially, this devaluation of the franchise is about more than just gerrymanders: indeed, the injury occurs even when the legislature acts without any partisan motives at all. It follows that districting that respects community is always constitutionally preferable to models that do not. We return to this point in Section III.

Finally, we note that computerization has made existing rules obsolete. So long as drawing maps was expensive, gerrymanders typically relied on crude tactics that lumped radically different groups - for example urban and rural voters - together. ${ }^{7}$ These were at least visually obvious. The difference today is that $21^{\text {st }}$ Century gerrymanders can afford to be subtle, adding up tiny changes to achieve decisive results. ${ }^{8}$ As a result, even rules that worked well in the past are no longer reliable.

## B. Law and Policy Before the Nineteen Sixties

The US Constitution's Elections Clause provides that states should "... regulate elections for Senators and Representatives, but that Congress may at any time by Law make or alter such Regulations." ${ }^{9}$ Despite this, prominent jurists doubted that Congress had the power to impose anti-gerrymandering rules well into the $19{ }^{\text {th }}$ Century. ${ }^{10}$ Beyond this, most Americans worried

[^2]that Congress did not know nearly enough to intervene. ${ }^{11}$ For this reason, the earliest antigerrymandering principles were pioneered in state legislatures. These notably included constitutional and statutory requirements that districts be geographically contiguous. ${ }^{12}$

Congress finally exercised its power to regulate districting in 1842. But while the statute represented a first attempt "to suppress the practice of gerrymander," ${ }^{13}$ legislators said almost nothing on this point. ${ }^{14}$ Instead, debate centered on whether states should be required to allocate each congressman to his own separate district. This was mostly seen as a fairness issue. Whigs pointed out that statewide districts could potentially disenfranchise up to $49 \%$ of each State's electorate. By comparison, individual districts would encourage the principle that "[t]he majority should govern but the minority be heard." ${ }^{15}$ While many Democrats probably agreed, they nevertheless fought the legislation, "not because they were opposed to the districting plan, but because they were unwilling to see it enforced on the States by the strong arm of Federal domination. ${ }^{16}$

The Act also followed earlier state laws by requiring that the new single member districts be contiguous. ${ }^{17}$ While there seems to be no legislative history on this point, scholars speculate

[^3]that the idea would have been obvious in any system that required separate districts. Regardless, the law's impact was fairly limited. While the new statute probably persuaded more states to adopt their own contiguity requirements, ${ }^{18}$ Congress continued to seat members from states that ignored the law. ${ }^{19}$ In 1872 Congress added still another safeguard by specifying that districts should also contain, so far practicable, an equal number of people. ${ }^{20}$

By then, many states were adding a fourth requirement that districts be "compact." ${ }^{21}$ Congress's 1901 Redistricting Act followed this lead by mandating districts based on "contiguous and compact territory." The House debate acknowledged that the new language was designed to prevent gerrymandering through "shoe-string districts," ${ }^{22}$ but worried that compactness was undefined and would invite challenges. ${ }^{23}$ Despite these worries, nothing was done to clarify the statute. ${ }^{24}$ Instead, Congress simply repeated all four requirements verbatim in 1911, ${ }^{25}$ only to drop them without comment in 1929. ${ }^{26}$ (The single member district

[^4]requirement was later restored.) ${ }^{27}$ Meanwhile scholars as late the Thirties continued to ask whether any of this legislation was constitutional. ${ }^{28}$ Given Congress's hesitancy, the idea that courts would step in would have seemed doubly remote.

Modern Case Law. The federal government returned to state election law with the Supreme Court's adoption of OMOV in Baker v. Carr (1962) ${ }^{29}$ and Congress's passage of the Voting Rights Act (1965) ${ }^{30}$. Despite this, the justices continued to steer clear of gerrymandering cases for nearly a decade until Gaffney v. Cummings (1973) ${ }^{31}$ rejected a lawsuit that had demanded state legislative districts approximating "... proportional representation of the two major political parties."32 Instead, the Court affirmed that the legislature was free to draw maps that afforded "any group or party ... a rough sort of proportional representation in the legislative halls of the State." The fact that this discretion "would have different political consequences" for different groups was irrelevant. ${ }^{33}$

Thirteen years later, the Court tried again. Davis v. Bandemer (1986) ${ }^{34}$ held that gerrymandering disputes were justiciable under the Equal Protection Clause. ${ }^{35}$ In theory this implied, inter alia, the justices' belief that "judicially discernible and manageable standards" existed to address disputes. ${ }^{36}$ The question was whether any standard could guarantee political fairness consistent with Gaffney's holding that the Constitution did not require proportional representation. Here, the justices explained that politics did not end on election day. This meant that fairness was normally available so long as legislators tried represent all citizens, including those who had voted against them. This was a broad hint that the Court would not interfere absent the kinds of abuses that had been practiced in the Jim Crow South, i.e.

[^5]"evidence of continued frustration of the will of a majority of the voters or effective denial to a minority of voters of a fair chance to influence the political process." ${ }^{37}$

O'Connor, Burger and Rehnquist concurred. Nevertheless, they voiced significant doubts that a standard could be found, emphasizing that any successful doctrine would have to respect "a strong and stable two-party system [which] has contributed enormously to sound and effective government." Absent this, Justice O'Connor wrote, courts would move
... toward some form of rough proportional representation for all political groups. The consequences of this shift will be as immense as they are unfortunate. I do not believe, and the Court offers not a shred of evidence to suggest, that the Framers of the Constitution intended the judicial power to encompass the making of such fundamental choices about how this Nation is to be governed. ${ }^{38}$

But for the moment, the Court did not have to decide. Instead, it remanded the case without ever saying what standard governed. Meanwhile three judges dissented, arguing that the existing suggestions lacked "...any clear stopping point to prevent the gradual evolution of a requirement of roughly proportional representation for every cohesive political group."39

The question remained what such a standard might look like. Shaw v. Reno (1993) ${ }^{40}$ opened up some possible directions by holding that a legislature's decision to follow "traditional districting principles" could rebut accusations that districts had been gerrymandered to achieve racial goals. ${ }^{41}$ Even though the standards were based on state rather than federal law, litigants could now potentially rely on traditional state redistricting criteria as evidence that whatever political bias had occurred ought to be tolerated. The question remained whether some or all of these grab bag principles could be reworked into a coherent doctrine.

The Court returned to the subject a decade later in Vieth v. Jubilirer (2004). ${ }^{42}$ Four justices argued forcefully that, despite eighteen years of effort, the lower courts had failed to "...shap[e] the standard that this Court was initially unable to enunciate" in Bandemer. ${ }^{43}$ Instead,

[^6]challenges had "almost invariably" been a waste of time, leaving the challenged gerrymanders in place. ${ }^{44}$ They then concluded that Bandemer had been wrongly decided and that partisan gerrymandering claims might not be justiciable after all. That probably would marked the end of the saga had it not been for a temporizing concurrence by Justice Kennedy that "in another case a standard might emerge." ${ }^{45}$

The Kennedy Boomlet. Kennedy's comment greenlighted still more gerrymandering litigation. The result was a multi-million dollar industry of computer-savvy consultants, high priced lawyers, and moonlighting social scientists. ${ }^{46}$ But they still had to develop an legal acceptable doctrine. This made Justice O'Connor's list of "traditional criteria" the natural place to start. While most were too narrow to support global rules, ${ }^{47}$ there were two possibilities. The first was visual standards: 22 states required contiguity while 17 required compactness. While the implementations differed widely in stringency, the concepts themselves were reasonably welldefined. The alternative was to build on the 13 states that required redistricting plans to maintain "communities of interest." ${ }^{48}$ Here, some scholars ${ }^{49}$ and States ${ }^{50}$ ignored the word "interest" and assumed that the term was identical to "community." In this view, "communities of interest" could be identified objectively by observing self-group identification and historical group identification. ${ }^{51}$ The alternative was for judges to grapple with a seemingly infinite number of ad hoc "interests" ranging from regional aviation industries to train corridors to
${ }^{44} \mathrm{ld}$.
${ }^{45}$ Vieth v. Jubilirer, supra note 42, 541 U.S. 267 at 312.
${ }^{46}$ Peter F. Galderesi and Bruce Cain, "Introduction: Redistricting Past, Present and Future," in Peter F. Galderesi, Redistricting in the New Millennium (2005) at p. 5.
${ }^{47}$ Examples included requirements that district preserve county and town boundaries ( 18 states); protect political incumbents (optional in 7 states, banned in 5); preserve the "core" of existing congressional districts (required in 7 states, optional in 3); and promote competition and/or avoiding partisan considerations (7 states). Royce Crocker, "Congressional Redistricting: An Overview," Congressional Research Service (Nov. 21, 2012) at p.2.
${ }^{48}$ Id.
${ }^{49}$ Kalyn M. Rossiter, David W.S. Wong \& Paul L. Delamater, "Congressional Redistricting: Keeping Communities Together? Professional Geographer 70 (4): 609-623 (2018) (developing computer algorithm that draws congressional districts based on culturally-defined indicators of community).
${ }^{50}$ For a state-by-state survey of the "community of interests" concept, see, e.g. Brennan Center for Justice, "Communities of Interest" (2010) https://www.brennancenter.org/sites/default/files/analysis/6\ Communities\ of\ Interest.pdf

[^7] Review 83(2): 461-92 (1997) at p. 477.
freeways. ${ }^{52}$ Since districts could not help splitting some interests even as they united others, ${ }^{53}$ this meant a nightmare jurisprudence in which judges would decide which "interests" would and would not be favored. ${ }^{54}$

The question remained how these traditional rules could be translated into formal computer algorithms. Here, visual standards based on compactness and contiguity had an enormous advantage: Mathematicians had studied similar tiling problems for centuries, so that the main challenges lay in comparatively well-understood issues like improving computational efficiency and accommodating additional criteria like OMOV. By comparison the "community of interests" approach was largely ignored. ${ }^{55} \mathrm{As}$ a result, the technical literature soon converged around a standard version of the problem defined by compactness, contiguity, and OMOV. ${ }^{56}$ The question remained whether the agenda could produce reasonable rules and, above all, whether judges would go along with them.

## C. Rucho Calls a Halt

[^8]The new methods came of age with Rucho's challenges to districting plans in Democratcontrolled Maryland and Republican-held North Carolina. Strikingly, all nine justices agreed that gerrymandering had occurred. Despite this, the majority declined to intervene. At one level, this simply continued the Court's long-standing reluctance to overrule state legislatures. The difference this time was that the majority's detailed objections seemed to rule out future challenges. Noting that the Court had never struck down a partisan gerrymander, the justices now asked whether such cases were justiciable at all.

The Conundrum of Fairness. The majority's most basic objection was that the challengers had failed to identify a workable concept of "fairness." If fairness was measured by outcomes, the only obvious focal point was proportional representation. ${ }^{57}$ Yet America's winner-take-all party system had never honored this principle, and - as Justice O'Connor Bandemer concurrence had stressed - would have been much less stable if it had. ${ }^{58}$ At the same time, procedural fairness standards seemed ad hoc and contradictory. Thus, grouping people with similar interests led to landslide outcomes in "direct conflict with criteria based on political competitiveness or electoral responsiveness." ${ }^{59}$ But engineering competitive districts was equally unacceptable since " . . . even a narrow statewide preference for either party ... [would] produce an overwhelming majority ... in the state legislature." ${ }^{60}$

Normally, the Court could have fled this indeterminacy by taking refuge in the Framers' supposed intent. The problem in this case was that all nine Justices agreed that originalism had nothing to say. ${ }^{61}$ The reason was that gerrymanders presuppose political parties - after all, why else would any politician "sacrifice" his seat for another? But the Framers had not anticipated parties, ${ }^{62}$ which seemed to make their thoughts and assumptions irrelevant.

[^9]Justiciability. Given the absence of any coherent standard, the Court had no choice but to reaffirm the legislature's broad discretion to implement fairness. ${ }^{63}$ Though the majority said little to explain this point, the observation nevertheless made sense. After all, "fairness" is not really about how votes translate into legislative seats - the real question is power. But in that case, legislatures already address minority rights through their rules governing debate, filibusters and supermajorities. Indeed, one of us (SM) has argued that is as it should be, and that legislators' rational self-interest provides a reasonable bulwark against majority tyranny. ${ }^{64}$ Given these broader issues, why not let legislatures decide electoral fairness as well?

Finally, the majority warned against the political dangers of having judges redraw congressional districts. "[E]ven when proceeding with best intentions," overruling legislative maps would saddle judges with "political, not legal, responsibility for a process that often produces ill will and distrust." ${ }^{65}$ Granted that the judiciary had sometimes tried to codify emerging social consensus in the past, ${ }^{66}$ those had been cases where the justices hoped to have a large majority on their side. By comparison, picking sides between Republicans and Democrats was certain to alienate half the country.

Dissent: The Rucho dissent angrily set out to show that Kennedy's hope has been fulfilled, i.e. that workable standards finally existed. Like Archimedes, this required both a lever and a place to stand. The lever, obviously, was computing. The place to stand turned out to be the assumption that individual precincts would go on voting for the same party forever. Comparing thousands of computer-generated maps against the actual one then became evidence of bias, with "[d]istance from the 'median' map ... indicat[ing] whether a particular districting plan harms supporters of one party to an unconstitutional extent." ${ }^{67}$

[^10]The argument is confusing not least because it splices an abstract probability measure ("distance from the median") onto what would normally be an exercise in plain language statutory interpretation. This makes it instructive restate the logic in more familiar terms. At bottom, North Carolina's formula defines a large-but-bounded catalog of lawful maps. Suppose that there are ten million of these. Then what the dissent is really saying is that the Legislature approved, say, 1,000 of these maps by mistake. ${ }^{68}$ But in that case, where should judges draw the line? The answer would be clear if there was an obvious discontinuity or "break" in the data. Then judges could point to some well-defined class of maps, and argue that it was vastly more extreme than all the others. But what happens if judges find themselves facing a bland continuum where each new map is only slightly more partisan than the one before? Then the only remaining option will be to compare the outliers against their personal intuition of what a "reasonable" legislator might accept. This seems hopelessly subjective. ${ }^{69}$

Worse, a standard which assumes that precinct data are immutable is hopelessly backwardlooking. After all, the only thing precinct data really tell us is how two, and only two, platforms competed against each other in the past. So long as platforms change slowly, we expect median maps to generate something like proportional representation. But when change accelerates, the maps are likely to change in large and unpredictable ways. Risk-averse politicians will avoid this gamble as long as they can. Yet democracy works best when political parties adjust their platforms to attract new supporters or make existing ones happier. Moreover, every platform will be archaic one day. A districting plan that encourages parties to be cling to outdated platforms can only make the next shift more wrenching when it does come. ${ }^{70}$ Historians tell us that there have so far been four "realignments" in which America's political parties radically changed coalitions. These can, as the example of the Civil War shows, be moments of great danger.

[^11]
## D. What's Left?

Rucho closes the door on conventional "wasted votes" methods, at least at the federal level. ${ }^{71}$ The question is what remains. We have argued that the opinion's main takeaways are that there is no identifiable theory of "fairness," that legislatures possess wide discretion, that courts should not legislate from the bench, and that no originalist theory of gerrymanders exists. These objections are connected. If there is no identifiable calculus of fairness or originalism, every map becomes an act of discretion. This simultaneously disables judges ${ }^{72}$ and vests legislatures with almost unlimited power. It is probably too late to overrule legislative discretion. But it is not too late to define its limits more rigorously, or invent a constitutionallydefensible safe harbor.

The nub of the problem, as the Court has said repeatedly, is finding some theory where the "gravitational pull" of proportional representation disappears. Here, O'Connor's "traditional criteria" offer two possible candidates. But two are potentially serviceable. The first is to tighten contiguity and compactness to the point where legislative discretion disappears entirely. This sidesteps the Court's "gravitational pull" objection by declaring that there is no center at all, or rather that the center is chosen at random. Section III asks whether this randomness can be achieved at acceptable cost. The second candidate sides with courts that write the word "interests" out of "community of interests" in favor of a standard that tracks actual communities. The strong version of this argument implies that this standard is constitutional. Granted that the Framers had no thought of gerrymandering, Section IV argues that they would have had very definite thoughts on elections and redistricting more generally. We explore this approach in Section V.

## III. Compactness and Contiguity

We have seen that visual standards like "compactness" and "contiguity" have a long history. Despite this, their connection to basic democratic concepts and the Constitution remains, in the words of one observer, "stunningly atheoretical."73 This Section fleshes out the most popular

[^12]explanation: That the arbitrariness of aesthetic standards is a deliberate effort to replace biased human decisionmakers with a random-but-neutral algorithm. This would supplant what Rucho had called the "gravitational pull" of proportional representation with a randomized but otherwise determinate algorithm. This puts us on familiar ground since law often adopts bright line rules to avoid hard factual inquiries. Like all bright lines, a successful rule need not be perfect. But it should at least block the most egregiously partisan outcomes most of the time. ${ }^{74}$ A final section examines the alternative possibility, that aesthetic standards are a highly imperfect heuristic for democratic values that the Framers would have agreed with.

## A. History and Law

Scholars have yet to unearth legislative history that would explain why $19^{\text {th }}$ Century state legislators invented contiguity and compactness standards. Instead, they have intuited answers in the usual way. Here, the most ambitious attempt is due to Professors Polsby and Popper, who posit that OMOV was designed to stop legislatures from setting up 19 single-voter districts and a $20^{\text {th }}$ district for everyone else. ${ }^{75}$ But in that case partisan legislators can usually achieve identical results by concentrating one party's votes in a single district so that its opponent wins the other $19 .{ }^{76}$ Contiguity remedies this defect. ${ }^{77}$ But contiguity can itself be evaded by reducing districts to a complex filigree which would sometimes be as "thin as telephone wires. ${ }^{\prime 78}$ Preventing this requires compactness. Even so, the argument fails to impress. If avoiding "telephone wire" districts were the only goal, almost any contiguity and compactness rule would suffice. If courts and politicians insist on stronger requirements, it must be that they have additional goals in mind.

Contiguity. The Supreme Court has never mandated contiguity ${ }^{79}$ and lower courts have generally enforced the requirement loosely. ${ }^{80}$ After-the-fact academic rationalizations argue

[^13]that the rule favors simple boundaries that incumbents, challengers, and voters can understand ${ }^{81}$; supports the appearance of fairness ${ }^{82}$; and provides an easily applied rule for the courts. ${ }^{83}$

Compactness. Scholars agree that contiguity by itself cannot prevent gerrymanders. ${ }^{84}$ Compactness requirements help fill this gap but vary widely in stringency. Five states define compactness by minimizing districts' allowable perimeters or areas, 16 specify shapes (e.g. rectangles), California weights compactness by population, Maine by travel distance, and 18 states offer no definition at all. ${ }^{85}$ For their part, scholars have suggested "almost a hundred" different measures ${ }^{86}$ without reaching consensus. ${ }^{87}$ In practice, compactness is just one among several factors; has steadily eroded since the Sixties ${ }^{88}$; and is in any case "... routinely ignored whenever more important stakes are on the table. ${ }^{89}$
recovered to the point where modern districts are considerably less contiguous than the historical norm. Micah Altman, "Traditional Districting Principles," supra note 12 at p. 179. Indeed, one federal court has even ordered non-contiguous districts to remedy a racial gerrymander. Dillard v. Town of Louisville, 730 F. Supp. 1546, 1548-49 (M.D. Ala. 1990).
${ }^{81}$ Royce Crocker, "Congressional Redistricting," note 47 supra at p. 11 and note 50.
${ }^{82}$ Id. The explanation is circular, claiming that contiguity is "fair" because it hinders gerrymanders without explaining why.
${ }^{83}$ Royce Crocker, "Congressional Redistricting," note 47 supra at p. 11 and note 51.
${ }^{84}$ Id. at p. 11 and n. 55 ("It has, however, been noted by several redistricting scholars that the contiguity requirement alone is not likely to discourage a determined "gerrymanderer.") and authorities cited therein.
${ }^{85}$ Michael McDonald, "The Predominance Test: A Judicially Manageable Compactness Standard for Redistricting" Yale L.J. Forum 18-43 (Aug. 7, 2019) at p. 22.
${ }^{86}$ Id. at p. 19.
${ }^{87}$ Roland G. Jr. Fryer and Richard Holden, "Measuring the Compactness of Political Districting Plans," 54 J.L. \& Econ. 493 at 494 (2011) (noting that " $[t]$ here is no consensus."); Vincent Cohen-Addad, Philip N. Klein and Neal E. Young, "Balanced Power Diagrams for Redistricting," (January 6, 2018) (quoting various definitions of compactness), https://arxiv.org/pdf/1710.03358.pdf; Stephen Ansolabehere \& Maxwell Palmer, "A Two HundredYear Statistical History of the Gerrymander," Ohio State Law Journal. 741-762 at pp. 745-47 (2016) (describing various proposed measures of compactness).
${ }^{88}$ Stephen Ansolabehere and Maxwell Palmer, "A Two Hundred-Year Statistical History of the Gerrymander," supra at 762: (finding "... a steady move away from geographic compactness as such a principle" toward "a different conception of representation in which compactness, although a standard, is valued little."); Michael McDonald, "The Predominance Test," supra note 85 at p. 20 ("No court has required a district to be redrawn solely for violating a state constitutional compactness requirement since 1981.")
${ }^{89}$ Daniel D. Polsby and Robert D. Popper, "The Third Criterion," supra note 73 at p. 340 and n. 184.

As usual, academic opinion has invented various post hoc rationalizations for the rule. These hold that compactness "correlates with (at least extreme) gerrymanders," is "ostensibly" neutral, ${ }^{90}$ and enforces a physical closeness that helps representative communicate with constituents. ${ }^{91}$ The question remains how effective it is. Older scholarship never claimed that it made gerrymanders impossible, only harder to design. ${ }^{92}$ This was no small thing when compiling a single map could take months, ${ }^{93}$ but matters much less in the age of computers. ${ }^{94}$

## B. Visual Standards as Mechanism Design

The most puzzling aspect of visual standards is the complete absence of any obvious connection to democracy or the Constitution. Vickrey's foundational paper ${ }^{95}$ explains why this might be good policy. It explains how determinate algorithms can grow maps starting from randomly selected tracts, ${ }^{96}$ removing "the human element... as completely as possible from the redistricting process. ${ }^{97}$ At the same time, the algorithm's randomness implies that traditional virtues like contiguity will occasionally fail. ${ }^{98}$ Here, we present a quantitative model of how

[^14]often this is likely to happen. If we find that the rule routinely fails in our simple example, we should expect it to fail even more frequently in life.

Baseline: Democracy Without Gerrymander. We begin by benchmarking the problem. How faithfully should we expect even a fair election to translate votes into congressional seats? To see the answer, consider that the US House of Representatives currently has 435 members. Assuming that voters are equally likely to identify as Democrat or Republican ${ }^{99}$, the final seat count is then identical to the result of 435 consecutive coin tosses. The laws of probability tell us that one side or the other should expect a 20 -vote majority (or better) roughly $3.4 \%$ of the time. ${ }^{100}$

Plainly, this is only a statistical fluctuation, and by no means the "will of the people." But what should really interest us is power. This means we should look for appropriate benchmarks in institutional rules like the US Senate's 60 vote supermajority. Here, the odds of either party capturing a supermajority by chance are $2.8 \%$. This seems like a reasonable risk, even if we expect the disaster to occur once every 200 years or so.

A Simple Gerrymander Model. The question is what rules like "contiguity" and "compactness" can do to stop gerrymanders from making the situation worse. Here, the best previous estimate is due to Prof. Taylor and coauthors, who calculate that a state consisting of twenty distinct geographic subregions can be divided into four districts of five subregions each in 24,310 ways. The good news, they add, is that contiguity and compactness reduce the number of possibilities for one real world test case (Newcastle upon Tyne, UK) to just 334 possibilities. ${ }^{101}$ But is that

[^15]good enough? To answer this, we need to go beyond Taylor et al.'s single example to something more general.

Consider, then, the maximally simple case of three adjoining congressional districts, each of which is perfectly square and contains 16 separate communities or "subregions" (Fig. 1). ${ }^{102}$ For the sake of definiteness, assume further that each individual community has a $50 \%$ ex ante probability of being majority-Democrat or majority-Republican. Now suppose that enterprising partisans try to redraw district borders so that two Democrats and one Republican are reliably elected to Congress. How effectively do our various visual rules allow them to


Fig. 1: A Simple Gerrymander do this for the baseline case where each of the three counties is exactly balanced between Republican and Democrat subregions?

Contiguity. The revised map must still be contiguous. ${ }^{103}$ This implies that a County A gerrymander must find at least one majority-Democrat subregion on County A's border with County B and likewise with County C. In practice, this is not a substantial impediment: We show in Appendix $B$ that politicians can find a suitable subregion $87.5 \%$ of the time.

OMOV. Assuming the subregions are equally populous, moving two majority Republican subregions would reduce District A's population below OMOV. Partisan legislators would then need to effect a compensating transfer of one majority Democrat region from Districts $B$ and another from District C back to District A. This turns out to be possible three-fourths (76\%) of the time.

Compactness. Does compactness stop these results? We consider two rules. Our stringent rule, which has never existed in any jurisdiction, define contiguity to require perfectly uniform, square districts. This would by definition make all changes to Fig. 1, including gerrymanders, illegal. However, this would also deprive legislators of discretion to adjust district lines to keep existing communities intact. On average, we expect district lines to slice off about $25 \%$ of each community they come in contact with.

[^16]By comparison, our moderate rule gives legislators enough discretion to adjust district borders to avoid splitting communities. We show in Appendix $B$ that the rule is still strong enough so that $82.4 \%$ of all districts can no longer be gerrymandered. ${ }^{104}$

The Downside of Random Solutions. Are these "bright line" rules good enough? Any final answer must ultimately depend on our tolerance for "acceptable" mistakes. All the same, the analysis is not encouraging. Despite being more stringent than any real-world rule, our "moderate" definition of compactness still permits up to $17.6 \%$ of all congressional seats to be gerrymandered. This is six times larger than our baseline estimate of natural fluctuations and seems excessive.

The question remains why no State has ever tried to make compactness strict enough to be determinate. Vickrey hints at the reason when he argues that it would be confusing to let congressional districts slice across preexisting towns and counties. ${ }^{105}$ The answer cannot be entirely right. After all today's legislatures routinely split districts to meet the Supreme Court's OMOV requirements. All the same, Vickrey has a point. The sin of gerrymander lies in moving citizens across district lines where their votes will have markedly less impact on the

[^17]congressional balance of power. ${ }^{106}$ Prof. Rossiter and her coauthors estimate that this happens in some states up to $17.1 \%$ of the time. ${ }^{107}$

## Visual Standards as Heuristic

So far we have followed Vickrey by assuming that visual standards are deliberately arbitrary. But in fact, most lawyers really do believe that odd shapes show bad intent. ${ }^{108}$ Here the most persuasive argument is still newspaper cartoonist Elkanah Tisdale (17681835)'s famous "gerrymander" sketch [Fig. 2].

The question remains whether Tisdale's argument is correct. At bottom, the argument has to be that long, skinny boundaries are objectively less natural. But if we ask why, Euclid's geometry does not help. Granted that "compactness" emerges more easily in some


Fig. 2: The Original "Gerrymander" (1815) coordinate systems than others, ${ }^{109}$ the choice itself is entirely arbitrary.

[^18]But in that case the intuition can only come from experience. And indeed, we often describe built-up areas in circularly-symmetric terms like "city centers" or "hubs." We can even offer the quasi-theoretical justification that people often settle near each other for social and business reasons. ${ }^{110}$ The trouble, once again, is that the rule is haphazard. Granted that noncompactness might reflect gerrymandering, it could equally be the product of history ${ }^{111}$, the geography of rivers, or of unbuildably steep hillsides. Finally, the compactness intuition works best for towns and cities. ${ }^{112}$ But the linear size of congressional districts varies 26 -fold. This makes the approximation a poor choice for many districts. ${ }^{113}$

Even so the intuition matters. We tolerate compactness because - sometimes at least - it really does trace community. But if that is the rationale it would be even better to eliminate visual criteria entirely. We seek a new method which is (a) equally determinate, but (b) replaces randomness with proxies that reliably respect real communities.

## IV. Community as a Constitutional Principle

The Framers said little about elections ${ }^{114}$, most likely because they saw no need to. They were, after all designing a limited government that could be spliced onto the states' existing elections. ${ }^{115}$ Even so, they would have had definite expectations. We argue that this included a clear understanding that district boundaries should coincide with actual communities.

## A. Community

We start from John Adams' observation that the purpose of elections was to produce "... an exact Portrait, in Miniature, of the People at large." ${ }^{116}$ The question then becomes what Adams

[^19]meant by "the People." Here he would surely have imagined the New England town meeting, whose membership was defined by the geographic borders of a specific community. ${ }^{117}$ This implies that "People" and "Community" are overlapping concepts. ${ }^{118}$

But in that case how should we define "community"? In our time the word "community" has been watered-down to describe almost any group that shares an occupation ("the legal community"), interest ("the hobbyist community"), or involuntary attribute ("the black community"). These however are only conditions that might encourage community - not the thing itself. Worse, we have seen that the number of potential interests is nearly infinite and contradictory. We think it would be much better to know whether citizens in a given area feel loyal to a collective whose decisions they are prepared to honor even when they themselves disagree.

In principle we could go out and ask. Survey responses, however, are often sentimental or dishonest. The best evidence is what economists call "revealed preferences." For community, this generally comes down to whether residents stay with the group or else leave at the first opportunity. Probably the most striking aspect of $18^{\text {th }}$ Century New England is how often community members faced this decision. On the one hand, citizens had to affirmatively volunteer to form new townships or join existing ones. ${ }^{119}$ On the other hand they stayed -

[^20]${ }^{119}$ Prof. Labaree has described in detail how volunteers came together to pursue a charter: "[A]bout 20 or 30 family heads ... could apply to the General Court for a grant of land... perhaps as large as six miles square." Thereafter the original proprietors were free to admit new settlers, though most closed their lands to further expansion after some sixty to eighty families had joined. Benjamin W. Labaree, "New England Town Meeting," American Archivist 25(2):165 (1962) at p. 165-166.
when they did stay - even though most townships were divided into distinct villages ${ }^{120}$ that could easily secede. ${ }^{121}$ This demonstrated community in the strongest possible way.

Alexis de Tocqueville's classic Democracy in America developed this experience into a formal theory of democracy. His central claim was that American government was built from individual communities of 2-3,000 souls each. ${ }^{122}$ These were simultaneously small enough to develop homogenous opinions yet large enough to be autonomous and self-sufficient ${ }^{123}$ For legal theorists, it followed that New England townships were both "sovereign" 124 and "primitive." ${ }^{125}$ For political theorists it rooted institutions to a preexisting communities where every member could participate ${ }^{126}$ and suppressed demagoguery by grounding politics in real human need ${ }^{127}$ and the "ordinary relations of life." 128

[^21]${ }^{122}$ Alexis de Tocqueville, Democracy in America Vol. 1 (Halcyon Classics n.d. [1835]) at p. 194.
${ }^{123}$ De Tocqueville reasoned that this scale ensured that "... on the one hand, the interests of its inhabitants are not likely to conflict, and, on the other, men capable of conducting its affairs are always to be found among its citizens." Id. at 193.
${ }^{124}$ Id. at p. 189 ("The village or township is the only association which is so perfectly natural that wherever a number of men are collected it seems to constitute itself.")
${ }^{125}$ Id. at p. 205 ("It is important to remember that they have not been invested with privileges, but that they have, on the contrary, forfeited a portion of their independence to the State...They are independent in all that concerns themselves; and amongst the inhabitants of New England I believe that not a man is to be found who would acknowledge that the State has any right to interfere in their local interests."); Id. at 215 ("England formerly governed the mass of the colonies; but the people was always sovereign in the township where its rule is not only an ancient but a primitive state.") By comparison, de Tocqueville considered larger Federal and State bodies as "second-rate communit[ies]" since they were incapable of supplying "the warmest of human affections, without arousing the ambitious passions of the heart of man." Id. at p. 210.
${ }^{126}$ Id. at 209 ("The New Englander is attached to his township, not only because he was born to it, but because it constitutes a social body of which he is a member.")
${ }^{127}$ Id. at 186 (Noting that township politics "... fulfill[] the ordinary duties and respond[] to the daily and indefinite calls of a community.")
${ }^{128}$ Id. at 211 ("But the township serves as a centre for the desire of public esteem, the want of exciting interests, and the taste for authority and popularity, in the midst of the ordinary relations of life; and the passions which commonly embroil society change their character when they find a vent so near the domestic hearth and family circle.")

The Way Forward. We have seen how Rucho struggled to find a coherent concept of electoral "fairness." A "community of interests" standard solves these difficulties at a single stroke. On the substantive side, community completely ignores the "gravitational pull" of proportional representation. This is no small thing, since the Court has stressed for decades that theories that favor proportional outcomes are unacceptable. Beyond that, community resolves the apparent paradox that neither "competitive" nor "safe" seats ensure "fairness." Instead, we see that this is a false choice: Communities, like families, sometimes agree ("safe" seats) and sometimes quarrel ("competitive" seats). The only thing that really matters is that they are allowed to decide as a group. Maybe the group will agree and maybe it won't. But that is best left to political parties rather than judges.

The deeper point is philosophical. Politics is a group activity. It follows that theories that reduce "community" to a sum of individual interests are badly incomplete. Put differently, many people have no opinion at all before they talk with friends, or if they do hold it so shallowly that they are prepared to reverse it. Indeed, it is precisely this flexibility that makes democracy possible. Nobody cares what New England town hall participants think before the meeting. The only thing that matters is what they agree on afterward. ${ }^{129}$

Of course, saying that community is fundamental is not enough. An acceptable legal doctrine must find some reasonably accurate and objective method for identifying communities where they exist. Modern social science has made good progress on this front.
B. Modern Research

The Framers imagined that Americans would elect representatives who would then meet in Congress and reason away their differences in the best Enlightenment way. The surprise, by de Tocqueville's time, was that the two party system let "The People" make binding choices before Congress ever met. Modern research has cast considerable light on how "social networks" -face-to-face discussions between voters - do this.

In practice there are two channels. The first is education, as voters share information and try to convince each other over backyard fences and office watercoolers. ${ }^{130}$ Crucially, this need not be a zero-sum game. Instead, participants can collectively come to ideas that no one imagined at

[^22]the start. ${ }^{131}$ When this happens, "The People" is more than the sum of its individual parts. It becomes an active and creative player in the country's politics.

But this is only half the story. Social forces also drive convergence. The reason, as de Tocqueville knew, is that Americans value political harmony at least as much as choosing the right policy. "[A]s long as the majority is still undecided, discussion is carried on; but as soon as its decision is irrevocably pronounced, a submissive silence is observed, and the friends, as well as the opponents, of the measure unite in assenting its propriety." ${ }^{132}$ Modern survey data confirms that voters holding minority views are indeed more likely to change their opinions or else abstain entirely. ${ }^{133}$ And if the urge to agree is strong enough, debate may become so truncated that the narrowing occurs almost at random. ${ }^{134}$ This explains why, in the words of

[^23][^24]one British scholar, "similar people vote differently in different places." ${ }^{135}$ Indeed, "neighborhood effects" turn out to be "just as large as class differences." ${ }^{136}$

Network Topology. Our understanding of voter networks is still in its infancy. ${ }^{137}$ That said, two inferences seem secure. The fact that opinion changes over short distances tells us that the underlying networks cannot be much larger than cities or counties. Additionally, the fact that debate can narrow opinion shows that the effective number of networks is fairly small. ${ }^{138}$
${ }^{135}$ Johnson et al. (2004); For a recent survey of the literature, see Russell Weaver, "Contextual Influences on Political Behavior in Cities: Toward an Urban Electoral Geography," Geography Compass 8(12) (2014) 874-91 at 877-79.
${ }^{136}$ Huckfeldt's pioneering study of Buffalo, New York voters found that in that working class respondents had a 0.60 probability of identifying Democrat if they lived in strong working class neighborhoods compared to 0.49 if they did not. Robert Huckfeldt, Politics in Context : Assimilation and Conflict in Urban Neighborhoods (Agathon Press 1986) at pp. 47-52. This fact is also consistent with the long-standing realization that careful statistical regression is based on socio-economic factors only predicts roughly $50 \%$ of party identification. See generally, Carol A. Cassel, "Predicting Party Identification, 1956-80: Who Are the Republicans and Who Are the Democrats?" Political Behavior 4 (3) (1982), pp. 265-28. Prof. Cassel showed that only about $50 \%$ of party affiliation can be predicted from factors like race, religion, social class, and attitudes about the role of federal government in the domestic economy. Id. at 278.
${ }^{137}$ We can imagine asking various questions. How many members do networks have? How are members connected to one another? Do some members have more links than others? Are some links more persuasive than others? How much geographic space do the various networks occupy?
${ }^{138}$ To see why, imagine an issue (e.g. tax rates) that can be summarized by a single continuous variable. Then we expect each network to converge on a different preferred value independently. It follows that a district with three networks would have three dominant viewpoints. On the other hand, a district with thousands of networks would splinter across every possible preference so that no narrowing at occurred. This would be indistinguishable from the usual mass politics model in which every voter chooses independently. See, e.g., Anthony Downs, An Economic Theory of Democracy, supra note 58.

The second key fact is that networks are ultimately defined by interactions between members, most of which are one-on-one or "dyadic." ${ }^{139}$ These basic building blocks have been exhaustively studied by Prof. Huckfeldt and his coauthors. ${ }^{140}$ [Fig. 3]. They find that the probability that a voter will interact with network members located distance " $d$ " from his home closely approximates an inverse square law of the form $\frac{K}{d^{2}}$, with half of all contacts occurring within a radius of 8.4 km ( 15 minutes' drive time) for nonkin. ${ }^{141}$ This information is crucial, since it lets us infer network strength - i.e. the expected number of voter interactions - from simple population densities. ${ }^{142}$ We refer to this as "Expected Link Strength" or "ELS" in what follows.

## C. Prescriptions



Fig. 3: Number of Dyadic Contacts (by distance, in miles).

We have said that voter networks are rooted in communities. But in that case what happens when districting maps split networks into two or more pieces? We argue that there are two distinct classes of injury.

The first relates to OMOV. Suppose that Network 1 falls entirely within District A, Network 2 falls entirely within District B, and Network 3 falls equally across both Districts. Then Network 3 will have systematically less influence than Networks 1 and 2, and this only because of where Network 3's members happen to live. This may not matter much to either political party,

[^25]provided that such accidents cancel over time. ${ }^{143}$ But it matters very much to the individuals who are disenfranchised. ${ }^{144}$

The second problem is that we rely on social networks educate voters and press them to adopt common views. It is reasonable to think that a network that spans two districts will split these efforts in half. This loss of information makes the vote less valuable to those who hold it, a fact which should in principle be visible as depressed turnout. This view is supported by evidence that citizens who live close enough to state lines to rely on out-of-state TV stations are systematically less likely to vote in their own state's elections. ${ }^{145}$

It follows that sound policy should draw district lines in a way that cuts as few dyadic links as possible. There are at least two mathematically sensible ways to do this:

Prescription 1: Maximizing Global ELS. We have already said that ELS scales as $\frac{K}{d^{2}}$ and is proportional the probability that two citizens will discuss politics as a function of distance. The simplest goal is to draw lines that reduce statewide ELS totals as little as possible. This would be easy without OMOV, because then all citizens (and their dyadic links) would find themselves united within a single super-district. Dividing voters across multiple, roughly equal districts makes this problem considerably harder.

The question, of course, is how we should weigh departures from OMOV against lost ELS. We start by noting that ELS and OMOV are both aspects of the same underlying right to vote. It follows that any balance should reflect how citizens value the relative impacts of ELS and OMOV on their franchise. In a perfect world, they would prefer their votes to be both undiluted (OMOV) and informed (ELS). The rub, as we have seen, is that OMOV and ELS are in tension. Ideally, the tradeoff would be set by observing what voters themselves value. ${ }^{146}$ Until then we
${ }^{143}$ This helps explain the Rucho majority's otherwise puzzling observation that individuals have OMOV rights but parties do not. Rucho, supra note 4 slip op.at p. 20 ("It hardly follows from the principle that each person must have an equal say in the election of representatives that a person is entitled to have his political party achieve representation in some way commensurate to its share of statewide support.")
${ }^{144}$ It is tempting to think that voters in Network 3 deserve protection less because their shared opinion is coerced or conformist. But of course the same is also true of Network 1 and 2 voters. In any case, the OMOV right cannot depend on how deserving individual voters might be. Then too, we expect social pressures to be generally mild and subconscious. Those influenced will generally consider their votes as thoughtful as anyone else's.
${ }^{145}$ Keena Lipsitz and Jeremy M. Teigen, "Orphan Counties and the Effect of Irrelevant Information on Turnout in Statewide Races," Political Communication, 27(2): 178-198 (2010) (citizens get their political news from crossborder news broadcasts are less likely to vote).
${ }^{146}$ The question is how voters compare their respective injuries from splitting and vote dilution. In principle, at least, the two effects could be disentangled by measuring voter turnout as a function of (a) how much districts depart from OMOV, and (b) how far voters live from the district's borders.
pragmatically adopt the Supreme Court's ipse dixit that district populations should ordinarily vary by no more than $10 \%$ and maximize ELS within that constraint.

Finally, we note that the same basic procedure can be extended to any other constraint that the Legislature values. For example, a Legislature that valued compact or historic districts could specify what percentage departure from its ideal was acceptable, and then require future redistricting to adopt whichever constrained map offered the highest ELS score. This has the virtue of honesty since the legislature would have to say exactly how much it valued the criterion compared to ELS. More immediately, a law that mandated the single best solution would end the Rucho dissent's central problem of choosing which out of millions of formally acceptable maps ought to be rejected. The winning procedure would also be highly resistant to covert gerrymanders. ${ }^{147}$

Prescription 2: Maximizing Community. So far we have assumed that policymakers should maximize ELS without regard to where it is located. But a voter who can already choose from 100 potential links will normally value the $101^{\text {st }}$ less than someone starts with no links at all. This suggests that maximizing global ELS might not capture each link's true value in very-low or very-high density communities.

Consider first a hyper-dense urban area. Even though citizens can theoretically access tens of thousands of links, most are unlikely to try more than a dozen or so. But in that case, the choice between districts that preserve an ELS of 2,000,000 and, say, 2,000,005 does little to improve voters' utility. At this point, our ELS goal reverts to Vickrey's mechanism design logic - an arbitrary standard that is useful mostly because it is determinate and neutral. This makes it tempting to preferentially save dyadic links in districts where they are scarce even if this reduces global ELS. ${ }^{148}$

Rural areas present the opposite problem. Here population density is so low that many citizens do not belong to any social network at all, ${ }^{149}$ while those who do will often reach out to distant

[^26]interlocutors. ${ }^{150}$ Once again, legislators could still employ the algorithm for its Vickrey benefits. That said, it might be more honest to identify under-dense regions at the outset and divide them by eye.

The silver lining is that these caveats seldom bite. For example, we expect determined voters to join networks down to a per capita ELS of 0.5 when it becomes physically impossible for the average voter to find an interlocutor. By comparison, Texas' Loving County, which ranks as the United States' second least populous country, has a per capita ELS of 202. Granted that there might may be emptier places in the United States, there cannot be many. ${ }^{151}$

## V. Testing the Algorithm: Texas

It is not enough to invent an algorithm. We also need to know that it produces reasonable maps and makes any tampering immediately obvious. We chose Texas as the most informative real world test because it offers large numbers of counties in geometries ranging from neatly stacked rectangles at Mid-State to outsized, irregular and/or tilted counties in the East, South, and West.

Because we treat counties as our basic "atoms," we ignore the six counties large enough to elect one or more congressmen entirely within their borders. ${ }^{152}$ This means allocating 244 counties with a calculated statewide ELS of 9.2 billion ${ }^{153}$ across 24 regions. Naively we would expect this partition to sever $23 / 24$ or $96 \%$ of all links. However, our inverse square law implies that the strongest pairs are generally close neighbors. This encouraged us to think that our algorithm could do better.

## A. Algorithm and Software

[^27]This section describes the software that one of us (AK) wrote to implement what the last section called "Prescription 1."

Competing Goals. Our goal is to aggregate granular "atoms" of location/population data into OMOV-compliant districts that maximize statewide ELS. ${ }^{154}$ Without OMOV the most efficient algorithm would go on merging whichever pairs offered the largest ELS until just one statewide super-district remained. Adding OMOV changes this solution in two ways. First, the most efficient algorithm now depends on picking whichever pairs offer the biggest per capita ELS until OMOV stops all further mergers at 24 districts. Second, we must now accommodate geometry. Once early-forming districts reach OMOV, further mergers become impossible. This can force late-forming regions into circuitous and even discontinuous shapes that reduce compactness and ELS.

Design Principle. We begin by noting that every possible algorithm can be described by just two rules. First, the algorithm must schedule the order in which atoms and later proto-districts merge. And second, it must decide which of all possible partners that atom or proto-district should merge with. We adopted the design principle that the first decision should take care of any geometric problems. This allows the second rule to revert to our maximally simple principle that whichever atom or proto-district is selected should always merge with whichever partner offers the largest ELS. ${ }^{155}$

This leaves the much harder question of what our first rule should be, i.e. how we should choose which individual atom or proto-district should merge next.

Three Experiments. To judge the size of these effects, we first tried an algorithm that ignored OMOV and geometry entirely. Instead, it simply collapsed our two rules by prioritizing whichever atoms or proto-districts offered the largest ELS. As anticipated, geometric effects occasionally intervened to violate contiguity and compactness. Even so, this first experiment performed remarkably well, preserving $15.6 \%$ of statewide ELS - nearly three times than our naïve 4\% prediction.

[^28]We next modified the algorithm to include Vickrey's suggestion that proto-districts should start at the State's borders and grow inward. More particularly, we selected the furthest county from Texas's geometric center ${ }^{156}$, allowed it to grow until it reached the Supreme Court's 10\% OMOV standard, picked the next furthest county among those that had not yet merged. We then repeated the process until OMOV stopped further growth everywhere. This yielded a compact map with only two continuity violations and increased the amount of preserved ELS to $17 \%$. We reproduce the resulting map nearby at Fig. 4A. While the map contains two discontinuities, the


Fig. 4A: Vickrey Districting implied error rate is so small that we expect any moderately large ensemble to contain at least one perfectly contiguous map.

[^29]Finally, we tried the inverse of our original approach by selecting the country with the least total ELS, allowing it to merge according to our Local Max rule, and then repeating the process until OMOV restrictions stopped further growth. The resulting map is reproduced nearby at Fig. 4B. This Min-Max solution was slightly less efficient Vickrey, changing reduced saved ELS to $15.8 \%$, and increasing the total number of contiguity violations from two to three. The experiment is nevertheless significant because it shows that maps with very similar ELS can look markedly different.

## B. Appearance.

Figs. 4A and 4B contain none of the grotesque shapes that Tisdale's cartoon satirized. To the


Fig. 4B: Min-Max Districting contrary: Neither looks obviously different from the maps typically drawn by legislatures. That said, the algorithm's districts hardly ever coincide with the State's actual ones. This is largely attributable to (a) the fact that even slight differences in district borders cumulatively add up, and (b) the Legislature's use of traditional criteria like deference to historic districts that our algorithms ignore. ${ }^{157}$

The biggest departure concerns contiguity. Fig. 4A features two breaks and Fig. 4B has three. Given that ELS respects community more efficiently than contiguity, this seems pardonable. This is especially true since contiguity in the real Texas map is often symbolic, sometimes connecting places through narrow corridors just a few miles across. ${ }^{158}$ That said, all of the

[^30]breaks occur in rural areas with small populations. This suggests that deliberate modifications could be introduced with little impact on our map's overall OMOV/ELS performance.

## C. Stability.

No algorithm can be better than its starting assumptions. We should therefore worry that partisans could manipulate the population ${ }^{159}$, geographic size ${ }^{160}$, shape ${ }^{161}$ and orientation of our "atoms" to push the algorithm toward gerrymandered outcomes.

We know, of course, that if politicians do enough tampering they can succeed. But large and widespread changes should be detectable. What really matters is that politicians cannot reach the gerrymander through subtle adjustments, or more precisely that small changes in inputs should produce (at most) only small effects. In general, proof will require repeated experiments in which "atoms" are randomly revised to see how final maps change in response. That said, there are at least three reasons to think that the algorithm is stable:

Large Scale Independence. Our ELS law strongly suppresses non-neighbor interactions above the county scale. Furthermore, our algorithms almost always hop-scotch across the state, so that the next proto-districts is often geographically removed from the one before. This implies that small changes at any one location are unlikely to influence outcomes beyond its immediate neighborhood.

We tested this logic by arbitrarily transferring 50\% of the geographic area and population of 4 counties to a randomly selected neighbor. This changed our final map in 4 out of 10 experiments. In two of those, the change was limited to moving one or both of the adjusted counties themselves. In the other two cases, 7-10 small rural counties were shifted. Because of their low populations, this would have carried negligible weight in the final vote tallies.

[^31]These numerical experiments suggest that the assignments are remarkably stable for large counties; small population counties are occasionally move, the changes appear uniformly unimportant. These observations suggest two additional ways to increase stability:

Atomicity. The surest way to suppress tampering is to reduce the individual size of the "atoms" on which the algorithm operates. Perfect implementation would require the algorithm to treat each resident as a separate "atom" of information. It follows that real districting systems should use the smallest possible building blocks. We recommend that this be done at the level of Census Bureau's census blocks ${ }^{162}$ or, at most, voting precincts. ${ }^{163}$

Small-Scale Blurring. Starting from smaller atoms would also ensure that early protodistricts were much smaller than the average interaction lengths defined by ELS law. During this period, non-neighbor interactions will greatly outnumber neighbor-toneighbor contacts. This is important since changes to any one district are much less likely to dominate a list of all pairwise interactions.

We tested this conjecture by rerunning Experiment 1 for rescaled counties whose physical dimensions and populations were comparable existing census blocks. ${ }^{164}$ Curiously, action at a distance seemed to improve compactness. ${ }^{165}$ Crucially, we found that the results were significantly more stable than our baseline experiment, particularly among medium-sized counties. ${ }^{166}$

[^32]
## D. Uncovering Gerrymanders.

Finally, we compare our algorithm results against the Texas Legislature's 2010 redistricting map. ${ }^{167}$ Here the only obvious and recurring difference is that the actual Texas map contains multiple elongated districts that connect major cities. Nine of these are enormous, stretching from roughly half to two-thirds the length of the State. ${ }^{168}$ The result, in each case, is to dilute urban Democrat votes at each end with large numbers of rural Republican votes in the middle. The pattern also carries over to six smaller inter-city districts. ${ }^{169}$ Our algorithms invariably block these outcomes by surrounding major cities with compact districts that track the roughly circular suburban communities.

## VI. Refinements and Extensions

The great advantage of our algorithm is that it uses an inverse square law to infer community (as expressed by ESL) from population in an especially transparent and simple way. This has the unexpected bonus of simplifying the tough computational problems that districting algorithms must overcome. ${ }^{170}$ That said, it remains possible that more complicated models could lead to still better estimates.

[^33]${ }^{168}$ The nine Super-Districts consist of District 11 running from Odessa-Midland to the outskirts of Austin and Ft. Worth, District 13 running from Amarillo to the outskirts of Dallas-Ft. Worth, Districts 19 joining Lubbock to the outskirts of Ft. Worth, District 23 running from El Paso to the outskirts of San Antonio, District 15 which runs in a long, skinny corridor from San Antonio to the Mexican border, District 23 which originates in El Paso and stretches to the outskirts of San Antonio. District 28. District 25 which stretches from Ft. Worth to Austin, and District 34 hugs the coast north from Brownsville before turning inland to San Antonio.
${ }^{169}$ Examples include Districts 5 and 6 near Dallas; District 10 which Ties Houston to Austin; District 12 which extends westward from Dallas to Ft. Worth; District 21 which ties Austin to San Antonio and large rural areas to the west; and District 35 , which ties San Antonio to Austin.
${ }^{170}$ Inverse distance laws save computation by letting algorithms focus on nearby pairs while ignoring the (much more numerous) distant ones. At least in retrospect, it is easy to see why algorithms based on community would include an inverse distance term. On the one hand, communities depend on interactions between people who live at different locations - i.e., they are extended objects. This means that traditional computer science approaches that rely on population densities and other point measurements miss a crucial aspect of the problem. On the other hand, the idea of community also implies limited size - for example, the idea of a "community" that simultaneously embraces both Dallas and Houston would be absurd. Taken together, these requirements imply

## A. Refinements.

Prof. Huckfeldt's data [Fig. 1] were collected in just two metropolitan regions and display obvious noise. This suggests that more and bigger studies can refine our ELS law. Beyond that, the inverse square relation almost certainly breaks down at short distances. After all, most of us wander freely within our immediate neighborhoods, and it is hard to believe that differences of a few feet would cause us to consult one neighbor more than another. Finally, link formation plainly depends on the cost of making physical contact. This suggests that average link distances are probably longer in rural networks and shorter in urban ones. ${ }^{171}$

In practice, we doubt that any of these corrections will significantly change our results. Granted that increasing the interaction scale would hasten the transition from our "action at a distance" to "nearest neighbor" regimes, no reasonable change will shorten the former to the point where it no longer blurs over the starting configuration of census block atoms. ${ }^{172}$

More Variables? Our method focuses on potential dyadic relations instead of actual ones. This makes sense, since the surveillance needed to track real interactions would be both costly and constitutionally suspect. ${ }^{173}$ That said, new proxy variables like party affiliation, race, income, and education would surely improve our estimates. The main objection to this, as we have said, is that constitutional standards should not favor some political platforms over another. No one would deny that white (rich, educated, etc. ...) voters are more likely to talk to each other than to outsiders. But that is no reason to draw districts that punish politicians from inventing new platforms that cut across these categories.

The more general principle is that supplemental variables should be as far removed as possible from the identity and economic categories that drive politics. We use headcount coupled with a distance law for political discussions. Prof. Rossiter et al.'s first estimate of community based on

[^34]place name data appears to be comparably neutral. Their second estimate based on social statistics (e.g. age, "female head of household") is more worrisome.

## B. Beyond Geographic Districts

Our algorithm efficiently divides states into geographic districts. We have argued that this captures the face-to-face interactions by which "The People" develop consensus opinions. But of course there are other ways to communicate, and we should be open to the idea that single member districts are not the only way to organize democracy.

At-Large Districts. Absent OMOV, our algorithm would go on merging atoms until only one state-wide district survived. This would eliminate the otherwise unavoidable tradeoff between ELS and OMOV. Indeed, some states actually do by elect congressmen from a single "at large" district. But in that case, why not make all districts "at large?" We have emphasized that voter networks fulfill an important function by narrowing opinion. But this only works when each district hosts, at most, a handful of networks. This is usually impossible for state-wide districts, although marginally feasible in small-population states. This may explain why the nation's atlarge districts are located in Alaska, Montana, the Dakotas, Vermont and Wyoming. ${ }^{174}$ The exception to this rule - Delaware - is equally instructive. The nation's second-smallest state occupies a roughly $25 \times 50$ mile rectangle. This geographic area is not much bigger than a typical community, and in any case comparable to many congressional districts across the US.

Internet Democracy. We have assumed that politics takes place through face-to-face contacts. But of course, the Framers knew that written correspondence can also persuade and inform, and telephones and the Internet have drastically expanded the possibilities. ${ }^{175}$ The attraction seems obvious: On the usual argument, Internet interactions are cheaper so that citizens can have more of them. ${ }^{176}$ Additionally, online networks might also encourage denser and more intricate network topologies that enhance education and narrowing.

But on-line communities also bring new dangers. So long as people choose places to live for non-political reasons, ${ }^{177}$ we expect the same basic mix of professions, ability, and education to

[^35]exist everywhere. ${ }^{178}$ Internet communities, on the other hand, are recruited by self-selection according to shared interests. As the Framers would surely have warned, this enormously increases the risk of faction. This is acceptable for party primaries ${ }^{179}$, but not general elections.

But in that case, who should be allowed to join an on-line congressional district? Assigning voters based on social data like profession or income would guarantee faction. ${ }^{180}$ On the other hand, limiting on-line communities to voters from the same geographic district would forfeit the Internet's signature advantage of bringing physically distant citizens together. Finally, letting voters choose between physical and on-line districts could well lead to overrepresentation of young and technically savvy voters on-line, producing a "natural gerrymander" that made it easier to elect extremists.

Despite this, some form of non-geographic community seems unavoidable, particularly if we think that most younger Americans have permanently shifted many of their face-to-face communities to on-line ones. ${ }^{181}$ The question is how to ground districts in on-line communities the way that New England townhalls grew from physical ones. The Facebook and Twitter monopolies are a good start: Today's on-line social media communities are very nearly as universal as physical ones. But the problem remains self-selection, i.e. the fact that the comparative handful of people who dominate the political "Twittersphere" are more extreme than the rest of us. ${ }^{182}$ One way to fix this is to notice that New England town halls routinely

[^36]${ }^{179}$ Italy's populist Five Star Party already submits major decisions to its on-line members. Davide Casaleggio, "How Italians Learned To Govern Themselves Through Technology," HuffPost (March 22 2017), available at https://www.huffpost.com/entry/five-star-movement-internet b 58cb008ae4b0be71dcf3048d.
${ }^{180}$ The practice of creating different legislative chambers for each profession has been tried many times, notably during the French Revolution and in Fascist Italy.
${ }^{181}$ AJ Agrawal, "Millennials Are Struggling With Face To Face Communication: Here's Why," Forbes (May 4, 2017), available at https://www.forbes.com/sites/ajagrawal/2017/05/04/millennials-are-struggling-with-face-to-face-communication-heres-why/\#6c47508a26e8.
${ }^{182}$ Matt Singh, "Is Twitter Distorting Public Debate?" CAPX (Oct. 5 2018)(Twittersphere is much more interested in politics and also "wildly unrepresentative in terms of its political views."), available at https://capx.co/is-twitter-distorting-political-debate/. Unless extremists are dense on the ground, the only place they can congregate is on-
fined citizens who failed to attend. ${ }^{183}$ This makes it natural to think that an on-line system could similarly condition the right to cast on-line ballots on subscribing to a particular newsfeed. ${ }^{184}$ Whether this would lead to real participation is unclear, but daily exposure to headlines might at least tempt members to join the discussion.

Finally, OMOV will eventually require multiple on-line districts just as it does in the physical world. By analogy with our ELS principle, we argue voters who communicate directly are separated by one "link," those connected by a mutual friend through "two links," etc. The goal of districting would then be to minimize the average number of links connecting voters across the district.

## VII. Theory into Practice

We saw in Section VI that maps with very close ELS/OMOV scores can sometimes look substantially different. This presents policymakers with a familiar choice. On the one hand, they can follow Vickrey by adopting whichever map performs best. Here the main wrinkle is that the number of possible permutations is so large that researchers may never be able to exclude the possibility that some other, undiscovered map might perform even better. We argue that State officials finesse this by publicizing their best-performing map for a period of, say, 180 days. If Democrat or Republican consultants can find a better map in this time, the State should adopt it. If not, the window should close even if other, better maps are later discovered.

But this is not the only possibility. Once a single best ELS map has been advertised and approved, policymakers could also let the legislature choose between all possible maps that save ELS within some fixed percent of this benchmark. That would create room to consider traditional criteria beyond OMOV and ELS, but also increase the chances of gerrymandering. ${ }^{185}$

Politics. We have argued that our algorithm is constitutionally motivated, transparent, and determinate. But none of this guarantees adoption. What are the chances?

[^37]The Rucho dissent ridicules the idea that legislators can ever reform elections they have a stake in. But is that justified? Certainly, the idea that incumbent politicians will never vote against their own interests is reasonable. But the dissent was thinking of a politics where many different maps were lawful. And this opens gave log-rolling maximum room to find maps that benefit every incumbent over his or her challengers. The here is that our algorithm, like Vickrey's, forces legislators to vote on a single take-it-or-leave-it map. This will randomly favor some incumbents and weaken others, implying a close vote on average. And if that is not enough, the average congressman only serves ten years. ${ }^{186}$ This means that political opposition can be diluted by phasing in the algorithm over a period years. ${ }^{187}$

Finally, our algorithm provides evidence of what a non-gerrymandered district map should look like. If nothing else, this will help voters to distinguish between accidental population patterns and those that show genuine tampering.

## VIII. Conclusions

Gerrymandering is malicious and wasteful, and Americans would get rid of it if only we knew how. For more than two centuries we didn't. Visual standards based on contiguity and compactness offered a reasonable stopgap before computers, but cannot be tightened at an acceptable cost. The only other potentially comprehensive standard is community.

The Rucho majority was right to declare that even the most sophisticated computational approaches are still inadequate. The only option now is a return to first principles. The Framers knew that community was the true basis of American democracy. They nevertheless lacked the computing power and social science to implement their vision. But today those gaps have long since disappeared. We can do better.

[^38]
## Appendix A: Democracy Without Gerrymandering

Following Vickery (1961) we analyze gerrymandering using a simplified checkerboard geometry. Specifically, we assume that the US is divided into 6976 communities. Each community has (a) identical population and population density as every other community, (b) identical (square) boundaries, and (c) is either a net-Republican or a net-Democrat with a $50 \%$ ex ante probability.

We further assume that some Vickrey-like algorithm has preliminarily allocated the communities into a random grid of 436 square congressional districts (CD) each of which contains 16 communities. Fig. 1b depicts three typical districts.

## Baseline: Democracy Without Gerrymandering.

 We seek the probability that a specific number of congressional seats will be assigned Republican or Democrat nationwide starting from an evenly divided population. We assume that the chances of a Democrat or Republican being elected in any single district is equivalent to the chances of a fair

Fig. 1b coin landing "heads" or "tails." This setup implies that outcomes can be calculated as the binomial probability,

$$
\binom{n}{k} p^{k} q^{n-k}
$$

where k is the number of "heads" ("Democrat") outcomes in n independent trials (congressional races), and $\mathrm{p}=\mathrm{q}=0.5$ is the probability of Democratic/Republican wins respectively.

We now seek the probability of electing a Congress in which one party enjoys at least a 20-vote majority over the other. Then $\mathrm{n}=435, \mathrm{k}=237$, which yields the cumulative probability (CP) of:

$$
P(k \geq 237)=P\left(k \geq\binom{ 435}{237} 0.5^{237} 0.5^{198}\right) \approx 0.034=3.4 \%
$$

Similarly, the chances of one party gaining at least a 60 vote supermajority in the Senate implies that $\mathrm{n}=100$ and $\mathrm{k}=60$. Then the CP is:

$$
P(k \geq 60) \approx 0.028=2.8 \%
$$

## Appendix B: Estimated Effectiveness of "Compactness" and Other Visual Rules

We now ask how easily Legislatures can create or increase partisan advantage within individual CD's by making small adjustments ("jogs") to initially square district borders. Our basic model continues to be the one previously described in Appendix A and Fig. 1B.

For simplicity, we concentrate on the case where three adjoining three CD's initially possess equal numbers of net-Republican and net-Democrat communities. We seek to estimate the probability that small jogs can be found that make District A net-Democrat and Districts B and C become net-Republican. As Fig. 1b shows, this means transferring one Democrat-leaning community from $A$ to $B$ and a second Democrat-leaning community from $A$ to $C$. We evaluate probability that such gerrymandering moves remain feasible under various constraints of contiguity, compactness, and OMOV.

Choice of Model. We assume initially square districts compromised of 16 indivisible and identically populous subregions or communities. ${ }^{188}$ This seems realistic on two grounds. First, the number is comfortably close to the average number of counties (15) in our calculated districts. ${ }^{189}$ Second, the average length of rural Texas counties is roughly 40 miles.

A single allowed operation is a " jog ", which is a simultaneous transfer of one community from one adjacent CD to another. The gerrymandering purpose of a jog is to create a net gain in the number of CD voting a certain way, which is why the minimal setup of the model is three CD's. In our case, a jog is required to be contiguous. For example, a transfer of $\{a\}$ to $C$ is a jog, whereas a transfer of $\{a\}$ to $B$ is not.

Contiguity. ${ }^{190}$ Contiguity implies that activists trying to achieve a successful gerrymander must simultaneously transfer one Republican element of the community set $\{\mathrm{a}, \mathrm{b}, \mathrm{c}, \mathrm{d}\}$ to C and one Republican element of the set $\{d, e, f, g\}$ to $B$. That would turn $A$ into Democrat, and $C$ and $B$ each into Republican, thus achieving a net gain of one Republican CD. For each of these two jogs the probability of at least one net-Republican community being transferred is:

$$
P(k \geq 1)=P\left(k \geq\binom{ 3}{1} 0.5^{2} 0.5^{1}\right)=P(k \geq 0.375) \approx 0.875
$$

Margin. Party discipline is imperfect, so that candidate personalities and similarly random factors can sometimes erase partisan advantage. Suppose party bosses decide that a gerrymander should lock in a $5 \%$ district-wide majority to suppress these fluctuations. ${ }^{191}$ Then moving two communities transfers $2 / 16=13 \%$ of the district's population. We conclude that

[^39]safety margins are not a binding in our model, though they would be if our modelled districts contained more communities.

OMOV. Our base model assumes that each county is equally populous, i.e. contains $1 / 16$ of the district's total population. After the jog we get 14 communities in $A$, and 17 in $B$ and $C$ each. Then moving a Republican community from District A to District B changes the districts relative populations by $17 / 14$, which is $21 \%$ - large enough to trigger the Supreme Court's requirement that district populations should normally differ by no more than $10 \%$. Restoring OMOV then requires partisan legislators to move one net-Democrat community from Districts B or C into District A. Further, the net-Democrat member cannot be adjacent to any net-Republican community previously moved to Districts B and C: Assume without loss of generality that the previously-moved Republican community is $\{a\}$ so that the non-adjacent net-Democrat community must come from the set $\{b, c, d\}$. The chances of being able to do this are then 0.875 . The joint probability of being able to do this in either District B or District C would then be $0.875^{2}=0.766$.

Compactness. We have remarked that more than 100 compactness rules have been suggested. Here we consider two, both of which are considerably stronger than any US jurisdiction known to us:

Rule 1: Strict Compactness. Maximal compactness would define $A, B$, and $C$ as squares while limiting allowable departures to scales far below the size of even one community. This would make all transfers in our model illegal, so that gerrymanders were ruled out entirely.

Rule 2: Moderate Compactness. Our second rule places a minimum thickness on lawful tendrils. In terms of our model it means that jogs can only operate on adjacent community pairs rather than individual communities (e.g, \{a\} could only be transferred as part of $\{a, b\},\{b\}$ as part of $\{b, c\}$, etc). ${ }^{192}$ This, however, would cancel the effect of transfers unless both districts supported the same party.

Knowing that District $A$ is partisan neutral overall, the chances of finding the same party affiliation for pair $\{a, b\}$ is $0.5^{2}=0.25$. The probability of finding one suitable grouping in three tries is then

$$
\binom{3}{1} 0.25^{2} 0.75^{1} \approx 0.42
$$

[^40]The probability for doing this simultaneously for both Districts B and C is then $0.42^{2}=$ 0.176. This means that $82.4 \%$ of districts, when restricted by moderate compactness, can no longer be gerrymandered.

Conclusion. Our moderate compactness rule reduces the number of permissible gerrymanders by approximately three-fourths. This makes it by far the most effective rule. That said, suppose that "only" one-fourth of all US districts are victimized by gerrymander schemes, and that these schemes generate a two-to-one majority for the gerrymandering party. Then this would still naively produce a nationwide majority of 36 seats in Congress. ${ }^{193}$ This is substantially larger than the expected binomial distribution value for a perfectly divided nation. It is also identical to the Democrats' 2019-2020 House majority.

## Appendix C: An ELS Optimization Algorithm

The base version algorithm is as follows.
For a set of N CBs we construct a $(\mathrm{N}-1)^{2}$ matrix of ELS, where each $\mathrm{N}-1$ vector records the pairwise ELS for all CBs in the set.

The algorithm runs until no unassigned CB's are left. At each step, it does the following:
Step 1. Selects the matrix element with max ELS. The vectors corresponding to that element are referred to as an adjoin pair candidate (APC).

Step 2. Compares APC's combined population against the defined limit. If the former exceeds the latter, records this and returns to Step 1.

Step 3. Performs a linear composition of APC by adding up ELS elements of APC and recording it as a new vector.

Step 4. Eliminates from the matrix the vectors which comprised the APC.
Matrix iteration shown in Fig. B.1. The values $g_{a b}$ denote the pairwise ELS for atoms $a$ and $b$. M-limit reflects the maximum allowed district population limit under OMOV

[^41]Let the following be defined:

1. M-limit $L$.
2. Pop function $M: r \rightarrow R$, where $r$ is regions.
3. Cumulative sum function $G: r \rightarrow R: G(r)=\sum_{i \in r} g_{i k}$
4. Regions $a, b, x, y$ with g-metrics table:

|  | $a$ | $b$ | $x$ | $y$ |
| :--- | :--- | :--- | :--- | :--- |
|  | 0 | $g_{a b}$ | $g_{a x}$ | $g_{a y}$ |
| $b$ | $g_{b a}$ | 0 | $g_{b x}$ | $g_{b y}$ |
| $x$ | $g_{x a}$ | $g_{x b}$ | 0 | $g_{x y}$ |
| $y$ | $g_{y a}$ | $g_{y b}$ | $g_{y x}$ | 0 |
| Set up. |  |  |  |  |

Let $\max (g)=g_{a b}$. Then $Q_{a b}=a \oplus b \Rightarrow M\left(Q_{a b}\right)=M(a)+M(b)$, and the iteration table looks as follows:

|  | $x$ | $y$ | $Q_{a b}$ |
| :--- | :--- | :--- | :--- |
| $x$ | 0 | $g_{x y}$ | $g_{a x}+g_{b x}$ |
| $y$ | $g_{y x}$ | 0 | $g_{a y}+g_{b y}$ |
| $Q_{a b}$ | $g_{x a}+g_{x b}$ | $g_{y a}+g_{y b}$ | 0 |

Iteration ( $a, b$ ).

Fig. B. 1

The algorithm's variants are defined solely by how the seed pair is selected. The Vickrey version starts from a randomly selected county, we select the most geographically distant county in the State as our seed. The algorithm then proceeds as defined above.


[^0]:    ${ }^{1}$ Ph.D in Mathematical Economy, Central Economics and Mathematical Institute (CEMI), Russian Academy of Sciences.
    ${ }^{2}$ Adjunct Emeritus Professor of Public Policy, University of California at Berkeley. The authors thank Eugene Bardach, Paul Seabright, Martin Quitt, Alex White, and the attendees of the 13th Digital Economics Conference in Toulouse, France (Jan. 9 - 10, 2020) for helpful comments. Any errors are ours alone.
    ${ }^{3}$ Available from the authors under Creative Commons License 3.0, available at https://creativecommons.org/licenses/by/3.0/.

[^1]:    ${ }^{4}$ See, e.g., Rucho v. Common Cause, 588 U.S. __ (2018), slip op. at p. 4.
    ${ }^{5}$ An even simpler type of gerrymander was already prevalent before the Revolution, when colonial governors drew legislative districts that either split hostile areas like Philadelphia so that they could not claim their suburbs' voting strength, or else centered districts on regions where the governor's friends where thick on the ground. See, e.g., Vieth v. Jubelirer, 541 U.S. 267 (2004) at 957 and authorities cited therein.

[^2]:    ${ }^{6}$ See discussion of Shaw v. Reno, 509 U.S. 630 (1993), accompanying notes 37-38, infra.
    ${ }^{7}$ The old-style districts are still being challenged. See, e.g., Justice Powell's description of Indiana's gerrymandered districts in Davis v. Bandemer, 478 U.S. 109 (1986) at pp. 162-64 and our own description of Texas's elongated districts in Section V.D, infra.
    ${ }^{8}$ As journalist Vann Newkirk remarks, "Gone is the era of elaborate cartographical sketches and oil paintings of salamanders, and of salted old-timer politicians drawing up their 'contributions to modern art' armed with markers and heads full of electoral smarts." Instead, today's gerrymanders have achieved "one of the holy grails of redistricting" by "microtargeting below precinct lines." Vann R. Newkirk II, "How Redistricting Became a Technological Arms Race," The Atlantic (Oct. 28, 2017). Newkirk adds that "While that kind of tinkering didn't dramatically reshape a whole lot of districts that had already been deeply gerrymandered by years of partisan mapmaking, the results of such micromanaging in the aggregate seem undeniable."
    ${ }^{9}$ US Const. Art 1, § 1.
    ${ }^{10}$ For example, Senator Daniel Webster wrote in 1832 that "If a State were to give one portion of her territory a representative for every twenty-five thousand persons, and to the rest a representative only for every fifty thousand, it would be an act of unjust legislation, doubtless, but it would be wholly beyond redress by any power in Congress; because the Constitution has left all this to the State itself." Quoted in Harold M. Bowman, "Congressional Redistricting and the Constitution, Michigan Law Review 31(2): 149-179 (1932) at p. 178 and note 97.

[^3]:    ${ }^{11}$ Id. (quoting Daniel Webster: "Whether the subdivision of the representative power within any State, if there be a subdivision, be equal or unequal, or fairly or unfairly made, Congress cannot know, and has no authority to inquire...")
    ${ }^{12}$ The earliest contiguity requirements were enacted in New Hampshire (1788), Mississippi (1831), and Missouri (1821). Micah Altman, "Traditional Districting Principles: Judicial Myths vs. Reality," Social Science History 22(2): 159-200 (1998) at pp. 168-70.
    ${ }^{13}$ See, Rucho, slip op. supra note 4 at p. 10 (citing E. Griffith, The Rise and Development of the Gerrymander 17-19 (1907)).
    ${ }^{14}$ Gerrymandering issues were barely discussed in Congress. Alabama Democrat George Houston appears to have made the only argument, warning that the districting provision could facilitate gerrymandering though he did not say why. The issue was equally neglected in the wider debate. The only comments seem to have come from a Cleveland newspaper which argued that the adoption of single representative districts would prevent gerrymandering, and an Ohio paper which warned that enlarged districts would facilitate gerrymandering. Personal communication from Martin Quitt to SM (e-mail dated Oct. 19, 2019 on file with the authors); see also Micah Altman, "Traditional Districting Principles," supra note 12 at p. 194 at note 21 (the only mention of gerrymandering in floor debates was "a hypothetical and hyperbolic rhetorical question asking whether if Congress could mandate single-member districts, it could not also mandate particular gerrymanders.")
    ${ }^{15}$ Martin H. Quitt, "Congressional (Partisan) Constitutionalism: The Apportionment Act Debates of 1842 and 1844," Journal of the Early Republic 28:627 (Winter, 2008) at p. 638. Small states often favored statewide districts because they allowed, for example, Alabama's united delegation to outvote New York's divided one. Id. The Whigs also stood to gain a modest number of seats if state-wide districts were outlawed. Id. at p. 637.
    ${ }^{16}$ Id. at p. 637 (quoting Rep. Thomas Arnold (W-TN)).
    ${ }^{17}$ Id. Act of June 25, 1842, ch. 47, 5 Stat. 491.

[^4]:    ${ }^{18}$ Micah Altman, "Traditional Districting Principles," supra note 12 at p. 168: Prior to 1913, only California and West Virginia made contiguity a constitutional requirement. Statutory requirements were passed in North Carolina (1868), Virginia (1902) Nebraska (1867) Texas (1876), Washington (1889), Wisconsin (1848) Arkansas (1868), Louisiana (1868), Massachusetts (1857), New York (1846), Utah (1896), and West Virginia (1873). Id. at pp. 169-70.
    ${ }^{19}$ Martin H. Quitt, "Congressional (Partisan) Constitutionalism," supran. 15 at pp. 650-51.
    ${ }^{20}$ Harold M. Bowman, "Congressional Redistricting and the Constitution, supra note 10 at p. 162.
    ${ }^{21}$ West Virginia's Constitution codified compactness in 1872. Statutory requirements existed in Rhode Island (1842), Nebraska (1867), Wisconsin (1848), Arkansas (1868), Missouri (1821), New York (1894), and West Virginia (1873). Micah Altman, "Traditional Districting Principles, supra note 12 at pp.169-70. Prof. Altman adds that at least one unnamed state constitution required compactness "as early as 1821," though the concept was "never formally defined." Id. at 172.
    ${ }^{22}$ Id. at $n$. 25 . Rep. Theodore Kluttz said that the phrase referred to the antiblack Mississippi congressional districts of 1883 and 1893. Rep. William Ryan further explained that "The legislatures in the Northern States are all Republican and if they were permitted to shoestring the districts, it would be impossible for the few Democrats from the Northern states you have to get here. It was put in by the Democrats on the committee, with the consent of the chairman of the committee, to show that he intended to be fair with the Northern Democrats." Id.
    ${ }^{23} \mathrm{ld}$. at 605 . Representative John Rixey of Virginia asked "who is to be the judge as to when districts are sufficiently compact?" and predicted it would "give rise to contests[.]" This marked the first time that compactness was ever debated in Congress, although Rep. Beltzhoover had complained on the floor in 1882 of "dumbbell" shaped districts and argued that contiguity and population equality were not sufficient to prevent this abuse. Id. at note 24.
    ${ }^{24}$ Micah Altman "Traditional Districting Principles", supra note 12 at pp. 172-73.
    ${ }^{25}$ Harold M. Bowman, "Congressional Redistricting and the Constitution," supra note 10 at 162.
    ${ }^{26}$ Id. at 163.

[^5]:    ${ }^{27} 2$ U.S.C. §2c (1967).
    ${ }^{28}$ Bowman, supra, note 10 at pp. 174-78 (reviewing legal uncertainties but ultimately concluding that Congress has broad authority).
    ${ }^{29}$ Baker v. Carr (1962) 369 U.S. 186 (holding that challenges based on one-man-one-vote principle to be justiciable).
    ${ }^{30}$ P.L. 89-110.
    ${ }^{31}$ Gaffney v. Cummings 412 U. S. 735 (1973).
    ${ }^{32}$ Id. at 738.
    ${ }^{33} / d$. at 754.
    ${ }^{34}$ Davis v. Bandemer (1986) 478 U. S. 109, 116-117 (1986).
    ${ }^{35}$ The question had been implicit in the Court's Gaffney decision but had not been addressed.
    ${ }^{36}$ Davis v.Bandemer, supra note 32, 478 U. S. at 123.

[^6]:    ${ }^{37}$ Id.at p. 133.
    ${ }^{38}$ Davis v. Bandemer 478 U. S. at 145 (O'Connor concurrence).
    ${ }^{39}$ Id. at 147.
    ${ }^{40}$ Shaw v. Reno, 509 U.S. 630 (1993).
    ${ }^{41}$ Id. at p.647. Racial gerrymanders were an easy case since courts could readily resolve them by ordering the legislature to compile a color-blind replacement plan. The difference for non-racial, partisan gerrymanders was that every plan favors one party or the other. This immediately threw judges back on the much harder problem of deciding how much partisanship was "too much."
    ${ }^{42}$ Vieth v. Jubilirer 541 U.S. 267 (2004).
    ${ }^{43}$ Id.

[^7]:    ${ }^{51}$ Stephen J. Malone, "Recognizing Communities of Interest in a Legislative Apportionment Plan," Virginia Law

[^8]:    ${ }^{52} /$ d. at p. 469.
    ${ }^{53}$ In re Legis/ative Districting, 475 A.2d 428, 445 (Md. 1984), appeal dismissed sub nom. Wiser v. Hughes, 459 U.S. 962 (1982) (fact that the number of potential communities is boundless and that no plan can represent all of them renders "communities of interest" concept "nebulous and unworkable."); Hastert v. State Bd. of Elections, 777 F. Supp. 634, 660 (N.D. III. 1991)(arguing that community of interests standard is so "subjective and elusive of principled application" that it is possible to construct a suitable argument for any incumbent in any district).
    ${ }^{54}$ By the early Nineties most observers seem to have concluded that the concept was "...too nebulous for principled application in apportionment plans" and even be on the edge of extinction. Stephen J. Malone, "Recognizing Communities of Interest in a Legislative Apportionment Plan," supra note 47 at pp. 479ff. Instead, Justice O'Connor's opinion gave it new life which continues to this day.
    ${ }^{55}$ One important exception is Prof. Rossiter and coauthors' efforts to formalize legal theories that equated "community of interests" with presently existing, real world communities. The work showed how community could be estimated from both culture (e.g. place name data) and sociodemographic variables. Kalyn M. Rossiter et al., "Congressional Redistricting," supra note 45.
    ${ }^{56}$ Professors Altman and McDonald describe six recent approaches, all of which focus on contiguity, compactness, and equal population. Micah Altman and Michael McDonald, "The Promise and Perils of Computers in Redistricting," 5 Duke J. Const. L. \& Pub. Pol'y 69 (2010) pp. 86-88 (surveying various recent algorithm papers). Among the most-commonly cited older papers, see e.g., Stuart S. Nagel, "Simplified Bipartisan Computer Redistricting," Stanford Law Review 17: 863-899 (1965) ("swap" algorithm balances population equality against population-weighted compactness); Henry F. Kaiser, "An Objective Method for Establishing Legislative Districts," Midwest Journal of Political Science 10: 200-213 (1966)(same); Robert L. Garfinkel and George L. Nemhauser, "Optimal Political Districting by Implicit Enumeration Techniques," Management Science 16(8): 495-508 (1970) (grading randomly generated districts by contiguity, compactness and equal population); and J.B. Weaver and S.W. Hess, "A Procedure for Non-Partisan Districting: Development of Computer Techniques," Yale L. J. 73:288308(1963)(recursively revising arbitrary districts to optimize equal population, compactness and contiguity).

[^9]:    ${ }^{57}$ We ignore the alternative, winner-take-all dictatorship possibility as implausible.
    ${ }^{58}$ Despite being accidental, America's two-party system gave the country a stable, centrist politics. This is usually explained in terms of the "median voter theorem," which holds that rational competitors in a two party system maximize votes by writing platforms aimed at centrists. See, e.g. Anthony Downs, An Economic Theory of Democracy (Addison-Wesley: 1957). It is an open question whether the theorem's assumptions still hold in the Trump era. Stephen M. Maurer, "The Healing Constitution," Kansas Journal of Law and Public Policy (2020: forthcoming), available at https://papers.ssrn.com/sol3/papers.cfm?abstract id=3361973.
    ${ }^{59}$ Bernard Grofman, "Criteria for Districting: A Social Science Perspective," 33 UCLA L. Rev. 77 (1985) at p. 90.
    ${ }^{60}$ Rucho, supra note 4 at p.18, quoting Bandemer, 478 U. S. at 130 (plurality opinion).
    ${ }^{61}$ Originalists can, of course, address unanticipated facts. This, however, depends on reconstructing the drafters' original theory of how the Constitution was supposed to work. The theory of two-party systems makes fundamentally different assumptions than the Framers.
    ${ }^{62}$ Modern scholars argue that the party system was not fully in place until the 1840s. Martin H. Quitt, "Congressional (Partisan) Constitutionalism, supra note 15 at p. 651 ("In 1842 House Whigs reduced the size of

[^10]:    their chamber for the first time. The lack of an uproar in the country showed "popular understanding of the new reality: The parties, not individuals, counted in Congress.")

    63 "There is a large measure of 'unfairness' in any winner-take-all system." Rucho, supra note 4 at p. 17. The Court did say that fairness issues can include "natural gerrymanders," where blind demographic forces concentrate one party's voters more than the other's. Id. at pp. 18-19.
    ${ }^{64}$ Legislators understand better than anyone that today's majority might easily be tomorrow's minority and have written rules sensible rules to protect both sides. See generally S. M. Maurer, "The Healing Constitution," Univ. of Kansas Public Policy Journal (forthcoming: 2020), preprint available at https://papers.ssrn.com/sol3/papers.cfm?abstract id=3361973.
    ${ }^{65}$ Rucho, slip op. supra note 4 at p. 15, quoting Vieth, supra note 42, 541 U. S., at 307 (Kennedy concurrence).
    ${ }^{66}$ Most famously in Roe v. Wade. For a definitive account, see generally Clarke D. Forsythe, Abuse of Discretion: The Inside Story of Roe v. Wade (Encounter 2013).
    ${ }^{67}$ Rucho, slip op. supra note 4 at p. 27.

[^11]:    ${ }^{68}$ To be sure, the theory has no future: In our computerized age, we expect modern legislators to find hyperpartisan maps first.
    ${ }^{69}$ We also note the majority's incredulity at the dissent's refusal to say whether future challenges must meet a 1-in-3,000 standard or something less. The criticism is deeper than it sounds. After all, the evidence says absolutely nothing about whether the challenged outlier would have turned up in the $3001^{\text {st }}$ iteration. But if it did, a $0.03 \%$ increase change in sample size would change the State's Republican delegation by $8 \%$. This is particularly troubling since most "compactness" definitions are ambiguous. This means that the clever experts can expand the 1-in-3,000 standard to something considerably higher any time they need to.
    ${ }^{70}$ The four American realignments are conventionally ascribed to the Jackson Era, the chaotic runup to America's Civil War, the Great Depression, and Nixon's rise in the Nineteen Sixties. Wikipedia, "Party Realignment in the United States, https://simple.wikipedia.org/wiki/Party realignment in the United States. The endless drama of the Trump/Resistance years strongly hints that similar shifts are once again underway.

[^12]:    ${ }^{71}$ This could change if liberals gain a majority on the Court.
    ${ }^{72}$ As the Supreme Court pointed out in Vieth, "'The judicial Power' created by Article III, §1, of the Constitution is not whatever judges choose to do ... It is the power to act in the manner traditional for English and American courts. One of the most obvious limitations imposed by that requirement is that judicial action must be governed by standard, by rule. Laws promulgated by the Legislative Branch can be inconsistent, illogical, and ad hoc; law pronounced by the courts must be principled, rational, and based upon reasoned distinctions." Vieth v. Jubelirer, supra note 42, 541 U.S. 267.
    ${ }^{73}$ Daniel D. Polsby and Robert D. Popper, "The Third Criterion: Compactness as a Procedural Safeguard Against Partisan Gerrymandering, Yale Law \& Policy Review 9:301 (1991) at p. 351 ("At the heart of any serious thought about gerrymandering lies a black hole in idea space. There is no generally accepted theory of representation that would allow one to specify what a legislature needs to look like in order to be worthy of its name. Democracy in this country has been vernacular, pragmatic, traditional, and, in its detail, stunningly a-theoretical.")

[^13]:    ${ }^{74}$ The gerrymander problem is particularly challenging for bright line rules. Supposing that just one hyper-partisan map turns out to be lawful, we expect legislatures to choose maximum partisanship $100 \%$ of the time.
    ${ }^{75}$ Daniel D. Polsby and Robert D. Popper, "The Third Criterion," supra note 73 at p. 328.
    ${ }^{76}$ Id. at pp. 328-329. Though they do not say so, the authors seem to assume that one party does not vastly outnumber the other.
    ${ }^{77}$ Id. at p. 330.
    ${ }^{78}$ Id. at p. 331. The authors present what they call a "rough analytic proof of this proposition" at note 144.
    ${ }^{79}$ Id. at p. 301; cf. In re Legislative Districting of the State, 370 Md. 312, 327-328 (2002) (US and Maryland Constitutions require that "the concepts of contiguity [be] fairly considered," though "not to insist that the most geometrically compact district be drawn.")
    ${ }^{80}$ Prof Altman reports that the first four decadal redistrictings all had at least one noncontiguous district, and all but one of the twelve redistictings between 1789 and 1913 contained at least one district where contiguity was questionable. While congressional contiguity requirements initially improved compliance, violations have steadily

[^14]:    ${ }^{90}$ Micah Altman and Michael McDonald, "The Promise and Perils of Computers in Redistricting," 5 Duke J. Const. L. \& Pub. Pol'y 69 (2010) at 92.
    ${ }^{91}$ Royce Crocker, "Congressional Redistricting," note 47 supra at p. 12.
    ${ }^{92}$ Writing in the Eighties, one leading gerrymander scholar asserted without proof that a compactness requirement would "almost certainly" rule out "the worst cases" and reported that many political geographers found it "quite difficult" to tamper with districts that were "perfectly regular" in appearance. Bernard Grofman, "Criteria for Districting: A Social Science Perspective," 33 UCLA L. Rev. 77 (1985) at 90 - 91. However, he added that many scholars thought that "the usefulness of requiring that districts be compact has been vastly overrated.") id. at p. 89; see also Vieth, 541 U. S., at 298 (plurality opinion) ("[P]acking and cracking, whether intentional or no, are quite consistent with adherence to compactness and respect for political subdivision lines").
    ${ }^{93}$ Vann R. Newkirk II, "How Redistricting Became a Technological Arms Race," supra at note 8 ("[M]apmaking was limited by computing power, the incredible burden of data management, the cost of hardware, the unwieldiness of computers, and the use of giant, slow map printers that literally drew maps with big markers" well into the digital era.)
    ${ }^{94}$ Daniel D. Polsby and Robert D. Popper, "The Third Criterion," supra note 73 at p. 332 and n. 148 ("Computers can endlessly crank out district plans which nevertheless conform to a fixed standard of compactness. Even under a constraint of compactness, an infinite number of district plans are still theoretically possible.")
    ${ }^{95}$ William Vickrey, "On the Prevention of Gerrymandering," Political Science Quarterly 76(1): 105-110 (1961). Vickrey won the 1996 Nobel Prize in Economics for "mechanism design," i.e. devising incentive schemes that suppress self-interested decisions. This explains his interest in gerrymandering and procedures for suppressing it.
    ${ }^{96}$ Id. at pp. 106-7.
    ${ }^{97}$ Id. p. 106.
    ${ }^{98}$ Id.at p. 107. Vickrey's algorithm discourages such outcomes by growing districts inward from the state's borders. Vickrey was willing to tolerate such errors, arguing "that on those rare occasions when an enclave does occur, the

[^15]:    resulting construction of an election district with non-contiguous parts will be tolerable as a freak for the sake of adhering to the straight forward rule." Id.
    ${ }^{99}$ The assumption is more than just an arbitrary benchmark. The "median voter theorem" predicts that rational competitors in a two-party system will always draft platforms that pander to centrists. Anthony Downs, An Economic Theory of Democracy, supra note 58. The ubiquity of close elections in American life shows the force of this model. A more careful analysis would adjust for the fact that each party is limited to a single, nationwide platform. This necessarily handicaps many regional candidates. For this reason, it might be more realistic to restrict our analysis to so-called "battleground" districts where the parties are evenly matched. As of 2019 there were 24 such seats. Cook Political Report, "House Race Ratings," https://cookpolitical.com/ratings/house-race-ratings.
    ${ }^{100}$ See Appendix A.
    ${ }^{101}$ Peter J. Taylor, Graham and R. J. Johnston in Bernard Grofman (ed). Electoral Laws \& Their Political Consequences (1986). While Taylor et al. do not mention compactness explicitly, modern maps of the city show precincts that would satisfy the concept in most US jurisdictions. See, e.g., Andrew Teale, "Local Elections 2007 Newcastle upon Tyne," http://www.andrewteale.me.uk/leap/map/2007/53/.

[^16]:    ${ }^{102}$ Our example is deliberately conservative since square districts have the largest ratio of interior to exterior subregions. Following the analysis presented in Appendix B, this implies that any other configuration always increases the chances for a successful gerrymander.
    ${ }^{103}$ For simplicity, we ignore districts that only meet at a single point. This does not fundamentally affect our results.

[^17]:    ${ }^{104}$ A recent paper by Professors Alexeev and Mixon similarly considers the efficacy of compactness rules in a state where each voter's location and partisanship are decided by the equivalent of a $50-50$ coin toss. They show that even very strict compactness rules cannot stop partisans from drawing districts that shift $70 \%$ of legislative seats to their own party. B. Alexeev and D. Mixon, "Partisan Gerrymandering with Geographically Compact Districts," Journal of Applied Probability 55(4): 1046-59. It is important to note that Alexeev and Mixon model let their hypothetic legislators draw districts anywhere in the state provided that OMOV and compactness are honored. By comparison, our analysis focuses on the more realistic case where legislators can only make incremental changes to districts whose basic locations and shapes have been predetermined by some combination of history and Vickrey-style algorithm. The fact that Alexeev and Mixon find visual standards so easily gerrymandered is nevertheless suggestive.
    ${ }^{105}$ William Vickrey, "On the Prevention of Gerrymandering," note 95 supra at p 107 (" $[\mathrm{N}] \mathrm{o}$ attempt has been made to take account of political sub-divisions such as towns, counties, wards, and the like. It is likely, of course, to be considered desirable to make election districts coincide as far as possible with other political boundaries.") Concern with existing political boundaries has noticeably declined since Vickrey's day because of efforts to meet OMOV. Royce Crocker, "Congressional Redistricting," at note 47 , supra p. 11 and n. 48 ("The political equality requirement led to many more split counties.")

[^18]:    ${ }^{106}$ Accord, Prosser v. Elections Board, 793 F. Supp. 859 (W.D. Wisc. 1992) at p. 863 (Posner, J: "[T]he achievement of perfect contiguity and compactness would imply ruthless disregard for other elements of homogeneity; would require breaking up counties, towns, villages, wards, even neighborhoods.")
    ${ }^{107}$ Kalyn M. Rossiter et al., "Congressional Redistricting," supra note 45. Significantly, Rossiter et al.'s simulations show that districts are typically much larger than communities. Id. This implies that many communities find themselves in district interiors where splitting is impossible.
    ${ }^{108}$ Legal scholars have been remarkably confessional on this point. See, e.g., Michael McDonald, "The Predominance Test: A Judicially Manageable Compactness Standard for Redistricting," Yale L.J. Forum 18-43 (Aug. 7, 2019) at p. 20 (the "'interocular test"" is "a scientific-sounding restatement of Justice Stewart's celebrated obscenity definition, 'I know it when I see it'"); Bernard Grofman, "What Happens After One Person-One Vote? Implications of the United States Experience for Canada," in Drawing Boundaries: Legislatures, Courts, and Electoral Values," p. 156 at 165 (John C. Courtney, Peter MacKinnon \& David E. Smith, eds., 1991) ("[T]he most powerful statistical test for partisan gerrymandering is ... 'Does the evidence for gerrymandering leap up and hit you between the eyeballs?"'"; Daniel D. Polsby and Robert D. Popper, "The Third Criterion," supra note 73 at p. 302 ("The diagnostic mark of the gerrymander is the noncompact district. Anyone who eyeballs a few legislative maps quickly will learn to recognize gerrymanders, although admittedly with imperfect accuracy.")
    ${ }^{109}$ Maps drawn in the familiar Cartesian "grid" coordinates do indeed favor compactness, in the sense that adding similar increments in the " $x$ " and " $y$ " coordinates reliably produce rounded droplet-type shapes. But Cartesian coordinates are a choice, and mathematicians often use alternative "axial" designations that pair directions with distances. Here the most natural shape is a long, skinny arc whose ends are maximally far apart - the very opposite of "compact."

[^19]:    ${ }^{110}$ Martijn J. Burger and Evert J. Meijers, "Agglomerations and the Rise of Urban Network Externalities," Regional Science 95(1): 5-15 (2016).
    ${ }^{111}$ The power of Tisdale's original cartoon lies in our incredulity that Massachusetts voters would ever willingly group themselves within a long, thin ribbon along the New York and New Hampshire borders. But it is equally true that America's coasts are thickly settled: Nobody would have looked twice at the same ribbon along the Atlantic.
    ${ }^{112}$ At the smallest and largest scales, our intuition breaks down completely. New York state is not centered on Manhattan, nor do Manhattan's dense neighborhoods obey any discernible symmetry. See, e.g., European Commission, "Global Human Settlement Layer," https://ghsl.jrc.ec.europa.eu/.
    ${ }^{113}$ Compare New York's $13^{\text {th }}$ Congressional District (10.25 sq. mi.) with New Mexico's 2d district (71,739.49 sq. mi.). Wikipedia, "List of United States Congressional Districts," https://en.wikipedia.org/wiki/List of United States congressional districts\#List of districts by area.
    ${ }^{114}$ Neither the Constitution nor, to the best of our knowledge, the debates around it speak to districting.
    ${ }^{115}$ The Constitution's Enumeration Clause [Art. I, §§1-2] provides little guidance, requiring only that census-takers record the existence of individuals and the State they live in.
    ${ }^{116}$ Letter from John Adams to John Penn dated March 27 1776. Available at http://www.masshist.org/publications/adams-papers/index.php/view/PJA04d039. The vision remained

[^20]:    controversial down to the 1840s, when Democrats called for a legislature that mirrored electors while Whigs advocated elite members who could moderate the worst features of democracy. Martin H. Quitt, "Congressional (Partisan) Constitutionalism," supra note 15 at p .631 , 635. In practice, neither party was particularly representative. Id. at 635.
    ${ }^{117}$ Karen Christensen and David Levinson (eds.), Encyclopedia of Community: From the Village to the Virtual World (Sage: 2003) p. 1395 ("All male residents were required to attend in $17^{\text {th }} \mathrm{C}$. Massachusetts on penalty of fine and all taxpayers could vote.")
    ${ }^{118}$ Except where "The People" includes two or more communities. Massachusetts colonists contemplating independence in the early 1770s routinely expanded their meetings to include outsiders from neighboring towns. See, e.g., "Old South Meeting House: Where the Boston Tea Party Began,"
    https://www.bostonteapartyship.com/old-south-meeting-house-history.

[^21]:    ${ }^{120}$ Settlements "were usually composed of several villages separated from each other by natural obstacles." David Syrett, "Town-Meeting Politics in Massachusetts, 1776-1786," William and Mary Quarterly 21(3) (1964), pp. 352366 at p. 353.
    ${ }^{121}$ Secession was common and often "... spiced by an ideological split or simply by a battle over the location of a new meeting house.) Labaree, "New England Town Meeting," note 119 at pp. 166-67; See also David Syrett, "TownMeeting Politics in Massachusetts, supra note 120 at pp. 352-366 at p. 354 ("But in most cases when disputes over the location of the meetinghouse arose, the town was simply divided into two or more towns.")

[^22]:    ${ }^{129}$ This view of community illuminates the Bandemer Court's insistence that political fairness often depends on elected officials' willingness to listen the citizens who voted against them. The community concept argues that minorities will find a voice so long as they can persuade just one of country's 435 congressional districts to agree with them. These seem like generous odds.
    ${ }^{130}$ This could include "bounded rationality" shortcuts in which voters adopt their opinions from friends or mainstream opinion on faith.

[^23]:    ${ }^{131}$ Our observation assumes that conversations across hundreds of thousands of citizens can occasionally produce ideas and compromises that politicians have overlooked. This seems reasonable in the same way that scholars have traditionally argued that the apparent perfection of European folklore reflects accumulated incremental improvements of innumerable storytellers and audiences. Bruno Bettelheim, The Uses of Enchantment: The Meaning and Importance of Fairy Tales (Knopf: 1976).
    ${ }^{132}$ Alexis de Tocqueville, Democracy in America, supra note 122 at p. 807. The phenomenon is similar to what economists call a "network effect," in which a desire for conformity encourages consumers to "tip" to a single standard.
    ${ }^{133}$ Ron Johnson and Charles Pattie, "Social Networks, Geography, and Neighbourhood Effects, in John Scott and Peter J. Carrington (eds.) The Sage Handbook of Social Network Analysis (2016) at p. 10 (detailed literature review shows that " $[T]$ he more supporters of a particular party an individual talks to, the more likely they are to switch their vote to that party if they previously either voted for an alternative or abstained.") available at https://pdfs.semanticscholar.org/924b/be8b9032620eab01d10010ca4b1aef542171.pdf

[^24]:    ${ }^{134}$ Sagalnik et al. repeatedly present the same 48 songs multiple on-line audiences, each containing thousands of listeners. The result show substantial randomness in which songs eventually became "hits," although song quality also plays a role. Matthew J. Salganik, Peter Sheridan \& Duncan J. Watts, "Experimental Study of Inequality and Unpredictability in Artificial Music Market," 311 Science 854 (2006).

[^25]:    ${ }^{139}$ We assume that conversations between three or more participants are comparatively rare.
    ${ }^{140}$ For the definitive account, see Brady Baybeck and R. Huckfeldt, "Urban Contexts, Spatially Dispersed Networks, and the Diffusion of Political Information," Political Geography (2002) 21: 195-220 at p. 198. The study reports the results of more than 3,500 interviews conducted in Indianapolis and St. Louis. The authors were able to identify 915 distinct dyads whose members discussed "important matters" or "government, elections, and politics." Id. at pp. 198-199.
    ${ }^{141}$ Brady Baybeck and Robert Huckfeldt, "Spatially Dispersed Ties Among Interdependent Citizens: Connecting Individuals and Aggregates," Political Analysis 10(3): 261-75 at p. 267 and Table 1. The corresponding figure for kin discussants was 6.4 km while the median distance between respondents and non-relative discussants is 5.7 km for non-coworkers and 9.2 km for coworkers. Brady Baybeck and R. Huckfeldt, "Urban Contexts, Spatially Dispersed Networks, and the Diffusion of Political Information," Political Geography (2002) 21:195-220 at p. 201.
    ${ }^{142}$ The existence of an inverse square law follows immediately if we assume that each voter's probability of making and maintaining contacts is inversely proportional to distance. Then the probability that one voter will find a suitable interlocutor is $(1 / r)$ and the probability that both parties will want to talk is $(1 / r)(1 / r)=1 / r^{2}$.

[^26]:    ${ }^{147}$ Partisan legislators could, of course, find some hyper-partisan solution and then select a cutoff that corresponds to their preferred map. Fortunately, this tactic would be unreliable. First, the legislation would require language ("No departures from perfect compactness greater than $83.452910 \%$ ") that made the fraud immediately obvious. Second, the Legislature could never be sure that it had, in fact, found the single best solution. If someone found a better map, the attempted gerrymander would collapse. Finally, populations evolve. Even if the gerrymander worked the first time, each succeeding redistricting would return a completely different - and approximately neutral - map.
    ${ }^{148}$ This could be done, for example, by making a special effort to preserve links between cities and their surrounding suburbs. This would, however, reopen the door to the familiar gerrymandering tactic of submerging urban areas in a sea of rural votes.
    ${ }^{149}$ For example, Texas' Loving County had 134 residents in 2017 , or about 0.2 persons per square mile. Wikipedia, "Loving County, Texas." https://en.wikipedia.org/wiki/Loving County, Texas. Baybeck and Huckfeldt found that

[^27]:    $20 \%$ of respondents did not belong to a network even in urban areas. Of those who did belong to network, onefifth (19\%) limited their contacts to relatives/ B. Baybeck and R. Huckfeldt, "Urban Contexts, Spatially Dispersed Networks, and the Diffusion of Political Information," supra note 141 at p. 201 and n. 5.
    ${ }^{150}$ People also rely on a significant number of long-distance relationships, especially but not entirely with family members. Ron Johnson and Charles Pattie, "Social Networks, Geography, and Neighbourhood Effects," supra note 133 at pp. 6-7.
    ${ }^{151}$ We have found no information on the marginal utility of ELS in high-density environments. However, the most reasonable guess is that voters are less willing to travel, so that our $1 / r^{2}$ law is reduced by some constant K . We address this possibility in the text accompanying note 171, infra.
    ${ }^{152}$ The excluded counties overlap Houston (Harris and Ft. Bend counties), Dallas (Tarrant), El Paso, San Antonio (Bexar), Austin (Travis). For this reason, our algorithm produces no counterpart districts to actual Districts 2, 7, 9, 16, and 18 (Houston), 20 (San Antonio), 22 (Fort Bend spilling over into parts of parts of Brazoria and Harris Counties); and 24, 26, 29, 30, 32, and 33 (Dallas-Ft. Worth).
    ${ }^{153} 9,185,067,597$.

[^28]:    ${ }^{154}$ The "atoms" could be census blocks, precincts, or entire counties. We picked the latter because the data are readily available and to minimize computational overhead.
    ${ }^{155}$ The choice is somewhat arbitrary. We cannot exclude that an ideal algorithm might sometimes require mergers that did not yield maximum ELS. We nevertheless conjecture is that the problem does not arise often, if at all.

[^29]:    ${ }^{156}$ McCulloch County. Our decision to use Texas' geographic center was arbitrary. As Prof. Vickrey notes, his "furthest county" prescription can be used for any randomly selected county. William Vickrey, "On the Prevention of Gerrymandering," note 95 supra at p. 107.

[^30]:    ${ }^{157}$ The fact that the Legislature occasionally draws lines within individual counties adds to the mismatch.
    ${ }^{158}$ For example, District 15 passes through four successive bottlenecks from the Mexican border to Austin. Each is only a few miles wide. This is not unusual. US Courts have approved corridors that narrow to highway bridges or

[^31]:    railyards or, in at least one case, disappear at high tide. See generally, Daniel D. Polsby and Robert D. Popper, "The Third Criterion," supra note 73 at p. 330 and note 142 and cases cited therein.
    ${ }^{159}$ For example, one can imagine dividing Texas into 36 super-counties - one for each congressmen - giving each a population equal to at least $51 \%$ of the OMOV cutoff. At this point all further mergers would be blocked. A similar if slightly more subtle option would to make 36 counties enormously more populous than the others. This would automatically ensure dominance of the emerging proto-districts.
    ${ }^{160}$ For example, legislators could suppress east-west districts by drawing narrow north-south counties that stretched from one end of Texas to the other.
    ${ }^{161}$ For example, legislators could tilt every north-south county so that it ran northeast to southwest. This would force any eventual North-South congressional district to undergo repeated jogs, lengthening its borders and cutting more ESLs.

[^32]:    ${ }^{162}$ As of the 2010 Census there were 11,155,486 blocks with an average population of 28 . Nearly half the blocks $(4,871,270)$ had a reported population of zero and some large blocks had hundred inhabitants. Wikipedia, "Census Block," https://en.wikipedia.org/wiki/Census block.
    ${ }^{163}$ Making atoms smaller than this scale would require precincts to present different ballots to different voters. This is already commonly done in North Carolina, Virginia and Texas. The Rucho plaintiffs argued that the Legislature had divided 563 of the State's precincts into more than 1,400 sections. Vann R. Newkirk II, "How Redistricting Became a Technological Arms Race," supra at note 8."
    ${ }^{164}$ This version of Experiment 1 adopted the assumption that census blocks are ten times smaller than real Texas counties and have an average population of one. Our rescaled size is consistent with the fact that "half the US blocks are smaller than a tenth of a square mile." Proximity One, "Census Blocks and Block Codes, http://proximityone.com/geo blocks.htm. This implyies an average size based on the formula $A=\pi r^{2}$ of 0.113 miles. Our population estimate is similarly based on the fact that there are approximately 11.2 million census blocks, implying an average population of thirty. We adjusted this downward to simplify the calculation and to reflect Texas' predominantly rural character.
    ${ }^{165}$ This was apparent both by eye and from the fact that the total number of districts formed within our OMOV constraint fell from 26 to 24.
    ${ }^{166}$ For purposes of comparison, we stopped the algorithm at the same point where our calculation for full-sized counties would have encountered the OMOV limit and stopped all further mergers.

[^33]:    ${ }^{167}$ The comparison is only possible because the Texas Legislature's districting maps keep most of the State's 254 counties intact. There are nevertheless exceptions where counties overlap two or more congressional districts. Thus, Wood County is split between Districts 1 and 5 ; Leon County is split between Districts 8 and 17; Lee County is split between Districts 10 and 17; District 12 overlaps parts of Parker and Tarrant Counties; Wise County is split between Districts 12 and 13 ; Floyd County is split between Districts 13 and 19; Chimney-like District 15 includes parts of Hidalgo and Wilson Counties; District 22 has non-Houston parts of Ft. Bend and Brazoria Counties; Gonzales County is split between Districts 27 and 34; San Patricio County is split between Districts 27 and 34; Upshur County is split between 1 and 4 ; and Brazoria County is split between Districts 14 and 22. Our list excludes counties whose populations are so large that OMOV requires two or more districts. See note _, supra.

[^34]:    some type of inverse distance relation, of which our $1 / r^{2}$ relation is among the simplest. The fact that compact shapes often maximize inverse-square law interactions also explains the otherwise mysterious existence of visual standards. That said, the inverse distance relation is more fundamental for any analysis that postulates community as a constitutional principle.
    ${ }^{171}$ Links may also depend on available road networks and travel time. Google routinely provides this information, although government might not trust the data enough to draw congressional districts.
    ${ }^{172}$ In principle, population count and locational data can be combined to form more complicated mathematical objects including density, gradients (i.e. the rate that density changes along a specific direction), a gradient-of-thegradient, and so forth. For now, there is no good empirical or theoretical reason to include these refinements. See also, Roland G. Fryer and Richard Holden, "Measuring the Compactness of Political Districting Plans," 54 J.L. \& Econ. 493 (2011) (arguing that future Voronoi analyses should include "more general notions of distance" that reflect roads and natural obstacles),
    ${ }^{173}$ Surveillance would be radically easier for congressional districts that were organized on-line, see Section VI.B, infra.

[^35]:    ${ }^{174}$ The fact that all of these states are rural suggests that per capita ELS may be too small to support networks in any case.
    ${ }^{175}$ As of 2005, face to face contact declines after five miles, telephone after 100 miles, and e-mail showed only minimal falloff with distance. Ron Johnson and Charles Pattie, "Social Networks, Geography, and Neighbourhood Effects," supra note 133, at p. 7.
    ${ }^{176}$ The case remains ambiguous if, as seems reasonable, web conversations are less persuasive than the face-toface kind.
    ${ }^{177}$ We exclude bureaucrats who choose to settle in the District of Columbia. The Framers' Generation anticipated this by denying the District seats in Congress.

[^36]:    ${ }^{178}$ The hypothesis that settlement patterns are apolitical is only approximate. There is good evidence that people seek out likeminded neighbors when moving and that this has reinforced geographic factionalism on net. Ron Johnson and Charles Pattie, "Social Networks, Geography, and Neighbourhood Effects," supra note 133, at p. 2. There are also numerous examples of highly motivated factions migrated for the express purpose of achieving a local majority. See, e.g., Megan van Frank, "'This Is the Right Place': Mormon Migration to Utah, The Spectrum (Feb. 21, 2015) (Mormon exodus), available at https://www.thespectrum.com/story/life/faith/2015/02/20/right-place-mormon-migration-utah/23771155/; Wikipedia, "Rajneesh Movement," (describing cult's attempt to win elections in remote Oregon community), available at https://en.wikipedia.org/wiki/Rajneesh movement; Madison Pauly, "Why Libertarians Are (Still) Plotting to Take Over New Hampshire," (libertarian efforts to create a local majority in New Hampshire), Mother Jones (Feb. 1 2016), available at
    https://www.motherjones.com/politics/2016/02/libertarians-new-hampshire-free-state/.

[^37]:    line. Hillary Clinton won the 2016 election 53-40 among those who posted about politics on Twitter and Facebook. "Most of this is a function of demographics. Our timelines are not weighted to contain the right proportions of different age groups, education levels, urban/rural geography, and so on." Id.
    ${ }^{183}$ Karen Christensen and David Levinson (eds.), Encyclopedia of Community: From the Village to the Virtual World (Sage: 2003) p. 1395 ("All male residents were required to attend in $17^{\text {th }} \mathrm{C}$. Massachusetts on penalty of fine and all taxpayers could vote.")
    ${ }^{184}$ Conditioning the right to vote on this modest effort is presumably constitutional give that twenty-one states still condition the right to file absentee ballots on some valid excuse for not participating in person. Vote.org, "Absentee Ballot Rules, available at https://www.vote.org/absentee-voting-rules/.
    ${ }^{185}$ The procedure would give a precise answer to the Rucho majority's taunt that there is no objective way to define permissible discretion, though policymakers would still need to specify the exact point where it becomes excessive.

[^38]:    ${ }^{186}$ Jennifer E. Manning, "Membership of the 115th Congress: A Profile" (Congressional Research Service: 2018) ("The average length of service for Representatives at the beginning of the 115th Congress was 9.4 years (4.7 House terms); for Senators, 10.1 years (1.7 Senate terms)."), https://www.senate.gov/CRSpubs/b8f6293e-c235-40fd-b895-6474d0f8e809.pdf.
    ${ }^{187}$ Incumbents who do not plan to retire can likewise use the algorithm's determinacy to predict and preemptively campaign among constituents who live outside their current districts.

[^39]:    ${ }^{188}$ Our choice of square districts is conservative. Allowing elongated shapes invariably lets more communities come into contact with neighboring districts, making it easier to form gerrymanders.
    ${ }^{189}$ Our median Texas district contains seven counties. This argues for model districts formed from nine subregions in a $3 \times 3$ pattern.
    ${ }^{190}$ As previously noted in text, the precise definition of contiguity remains uncertain. We therefore ignore legally controversial diagonal "point contacts" between districts in the interests of simplicity. Including them would not materially change our conclusions.
    ${ }^{191}$ Readers can confirm this by viewing county level returns for the 2018 Cruz-O'Rourke Senate race in Politico, "Texas Senate Election Results 2018," https://www.politico.com/election-results/2018/texas/senate/.

[^40]:    ${ }^{192}$ Three is inadmissible since it would create a "reverse tendril" among subregions that remained part of their original district.

[^41]:    ${ }^{193}$ We have argued that even a maximally stringent standard would let politicians gerrymander roughly one-fourth of all counties. This is enough to put 109 of the 435 House seats in play. If these seats were all gerrymandered on our 2:1 pattern, we would expect tampering to create a 36 -seat majority - a substantial majority compared to random fluctuations. This would, of course, be reduced to the extent that some state legislatures gerrymandered in favor of Democrats while others sided with Republicans. Additionally, the existence of nationwide platforms implies that most local races lean toward one party or the other. A better estimate would focus more narrowly on the country's 24 tossup seats. See, e.g., Cook Political Report, "House Race Ratings,"
    https://cookpolitical.com/ratings/house-race-ratings.

