Lecture: Taxes and Labor Supply

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EC230

Outline of Lecture:

1. Basic labor supply model with linear budget set
2. Adding taxes to budget set
3. Empirical literature


E. Saez “Do Taxpayers Bunch at Kink Points?”
Motivation and empirical regularities

- Main lesson from optimal tax literature: optimal tax rate depends inversely on compensated wage elasticity of labor supply
- In this lecture we will discuss issues around the estimation of this important parameter
- Enormous literature in public finance and labor
- Different measures of labor supply:
  - Extensive margin (work or not)
  - Intensive margin (hours of work)
  - Retirement, transition into retirement
- We will also discuss the elasticity of taxable income as a more general characterization of labor supply: tax avoidance, old Feldstein point that there are other margins then hours of work
- Our plan:
  - Lay out basic static model, one earner and two earner
  - Note role of taxes
  - Discuss alternative approaches for identification
  - Discuss Eissa, Saez
  - Labor supply and low income population
  - Labor supply and high income population
Static Labor Supply with Linear Budget Constraint

Individual faces exogenous deterministic wage (w) and price of other goods (p). They also may receive non-labor income (N). Utility is a function of leisure (ℓ) and other goods (x). The choice problem is:

Maximize $U(\ell, x)$ subject to $wh + N = \rho x$

$h = T - \ell$

Where:

- $w$ = hourly wage
- $h$ = hours worked ($\ell=$leisure)
- $T$ = time endowment
- $x$ = composite commodity
- $\rho$ = Hicksian price index ($\rho x$ can be replaced by total income $y$ or consumption $c$).
- $N$ = non-labor income

Maximize $U(\ell, c)$ subject to $wh + N = y$

$h = T - \ell$

(Usual) Assumptions:

- Increasing in $l$ and $y$ (decreasing in $h$)
- Leisure and consumption are normal goods
intercept = \( wT + N \) (full income at full hours)
slope = \(-w\) (loss in income of one more hour of leisure)

Therefore indifference curves have usual shape. We are typically interested in studying the determinants of **hours worked** but we model the determinants of leisure and then translate back to hours.

Deriving budget constraint:

\[
y = wh + N, \quad h = T - \ell
\]

\[
y = w(T - \ell) + N
\]

\[
y = (wT + N) - (w\ell)
\]
First Order Conditions (for interior solution)

\[ wU_c + U_h = 0 \]
\[ w = -\frac{U_h}{U_c} \]

Maximizing \( U \) with respect to \( h \) yields the labor supply function

\[ h = h^* (w, N) \]

**Corner Solution**

Define \( w^* \) = reservation wage
\[ = -\frac{U_h}{U_c} \text{, evaluated at } h=0 \]

\( w > w^* \) then work \( [h > 0] \)
\( w < w^* \) then no work \( [h = 0] \), equivalent to \( h^* < 0 \)
Comparative Statics:

Uncompensated elasticity of labor supply \[ \varepsilon^u = \left(\frac{w}{h}\right) \frac{\partial h}{\partial w} \]

substitution effect \(<0\)
income effect \(>0\) (if labor normal)
Can be positive or negative (backward bending labor supply)

Income effect parameter \[ \eta = w \frac{\partial h}{\partial N} \]

If leisure is a normal good, then negative
(Imbens, Rubin, Sacerdote AER 2001)

Compensated elasticity of labor supply \[ \varepsilon^c = \left(\frac{w}{h}\right) \frac{\partial h^c}{\partial w} \]

Always positive
Slutsky equation:
\[
\frac{\partial h^c}{\partial w} = \frac{\partial h}{\partial w} - h \frac{\partial h}{\partial N}
\]

Implies \( \varepsilon^c = \varepsilon'' - \eta \)

\( \varepsilon^c > 0, \eta \leq 0, \varepsilon'' < \varepsilon^c, \varepsilon'' \Leftrightarrow 0 \)

\( \varepsilon^c \) is the most important parameter because it measures the cost of distorting wages using taxes

Elasticity of participation:

Positive, increase in wages leads to an increase in participation (no ‘income effect' when considering extensive margin)
Adding taxes to labor supply model

Example #1: a uniform, proportional tax denoted as t

Max $U(\ell, y)$ s.t. $w(1-t)h + N = y, \ h = T - \ell$

FOC:
$$-\frac{\partial h}{\partial U} = w(1-t)$$

LS function: $h = h^*(w(1-t), N)$

Observations:
1. Net of tax wages belong in the labor supply equation (not gross wage)
2. Policy question: How do taxes affect hours worked?
   Elasticity can tell us; theory does not even tell us the sign!
3. How do taxes affect labor force participation?
   taxes $\rightarrow$ reduction in net of tax wages; no change in reservation wage
   $\rightarrow$ probability of work decreases
The empirical labor supply literature has evolved from looking only at hours worked to focusing on labor force participation:

- Clear predictions of theory
- Hours harder to model, perhaps not free to choose
- Participation margin more elastic than hours margin
- Increasing focus on more elastic labor supply groups (e.g. women) where participation is important.
Example #2: Progressive marginal tax rates
Consider three, increasing, marginal tax rates. Budget constraint becomes:
\[ Y = wh + N - \text{Taxes}(wh, N) \]

Given that utility function is concave, and budget set is convex, then we know there is a unique tangency (or corner solution) on one of the segments.

Possibilities: not working \((h^* < 0)\)
tangency on 1\(^{st}\), 2\(^{nd}\) or 3\(^{rd}\) segment
on a kink (expect people to be bunched on convex kinks)
Consider someone on the highest segment

First Order Condition:

\[-\frac{\partial U}{\partial h} / \frac{\partial U}{\partial y} = w(1 - t3)\]

Which implies the labor supply function:

\[h = h^* (w(1 - t3), Y^v_3)\]

Or, more generally:

\[h = h^* (w_n, Y^v)\]

where \(w_n = \) net of tax wage
\(Y^v = \) virtual income

Determining participation: \(h^* < 0\) then no work
\(h^*(w(1-t1),N)<0\)
Suppose you want to empirically implement this model with some data set. You use a linear hours of work equation and relate hours worked to net wages and net non-labor income (virtual income):

\[ h = \alpha + \beta w_n + \delta Y_v + \gamma Z + \varepsilon \]

**KEY OBSERVATION:** Labor supply theory tells us that the labor supply equation is a function of net wages and net non-labor income. Yet these are themselves a function of hours worked. Endogeneity.

We will see LOTS of examples this quarter of other government programs that change the budget set rendering net wages endogenous.
Empirical issues with identification of labor supply elasticity:

Data:
Current Population Survey (annual, starting in 1960s)
Panel Study of Income Dynamics (panel, 1968-)

Starting with basic OLS framework:

\[ h_i = \alpha + \beta w_i + \gamma N_i + \phi X_i + \varepsilon_i \]

Pencavel 1986 survey for men: \( \varepsilon^u = 0, \ \eta = -0.1, \ \varepsilon^c = 0.1 \)

Killingsworth and Heckman 1986 survey for women:
Much larger elasticities with larger range (0 to over 1)
Identification issues:

1. Cross-sectional identification of \( w \), high wage guys have more taste for work independent of wage?

2. Measurement error: wage usually computed as earnings divided by hours. Spurious negative correlation.

3. Measurement of hours itself: is "labor supply" just hours worked?

4. Functional form sensitivity: linear model means elasticity is \( \frac{w}{h} \beta \). Varies across persons. How to compute average elasticity? How sensitive is elasticity to functional form?

5. Taxes. Early literature ignored taxes. Theory tells us that equation should depend on net of tax wages and nonlabor income. Taxes generate endogeneity of net wages.

6. Non-participation: OLS regression can only be run for those working because there is no wage for those not working. But participation is (obviously) correlated with tastes for work and is an important issue for women, especially married women. Result is that basic estimate will miss extensive margin, which may be the largest margin of response.
Empirical Estimates

*Negative income tax experiments*
NIT conducted in late 1960s/, early 1970s in Seattle/Denver (SIME/DIME) and other rural sites in the US. Basic design of the program was a lump sum transfer with a 50-70% phaseout rate.

Basic result was significant labor supply response, but small.
- Men (elas small, 0.1)
- Women (elas 0.5), concentrated on extensive margin

This early attempt at experimentation in the US was not ultimately successful. Experiments were poorly designed.
-- nonrandom selection into experiment (selected on income)
-- nonrandom assignment to T and C groups
-- self reported earnings with incentives for T to underreport so that they got NIT payment (Lesson: need to match to administrative records: UI, SS, firm tax records.)
-- sample attrition

Nonrandomness undoes the simple T/C comparison that is so powerful in randomized studies. So much statistical modeling was used here.
Hausman nonlinear budget set model

There was a lot of attention to taxes and non-participation in the late 1970s early 1980s:

Structural nonlinear budget set models (Hausman)
Sample Selection methods (Heckman)

Idea of Hausman model:

- Wanted to identify deep parameters
- Assume functional form for preferences (usually linear hours equation), specify where observables and unobservables enter the model
- Specify budget set
- Solve algorithm for maximizing utility, ranges of unobservable that imply location on kinks and budget segments
- Find estimates with maximum likelihood methods
- Hausman 1981 results: large compensated elasticities (due primarily to large income effects). Larger elasticities for women.
- Very influential and method applied a number of times
Criticism of Hausman model:

- Sensitivity to functional form
- Does not address issue that wages are correlated with tastes for work

Static labor supply model predicts that individuals should bunch at the kinks in the tax schedule. Little evidence that they do.
Saiz “Do Taxpayers Bunch at Kink Points?”

-- Basic prediction of kinked budget constraint model is that we should see people bunched at the convex kinks. (And we should see a gap in the distribution at nonconvex kinks.)
-- Some papers have examined particular applications (social security earnings test, welfare recipients around notch, WFTC and hours restriction) but no study has examined this among taxpayers in US.
-- Simple, clever paper using the best data (tax data)

Modeling insights
• Less curvature in indifference curves (higher substitution elasticity) $\Rightarrow$ more bunching
  $$dz*/z* = e \frac{dt}{1-t}$$
  e=sub elasticity, t=MTR, z=taxable income
• Therefore if there is little evidence of bunching (and model is valid) $\Rightarrow$ small elasticity of taxable income
• Later he considers changes to model to explain lack of bunching (uncertainty in income, constrained hours choice)
Data
• IRS annual cross-section of taxfilers (1960-1997) N=100,000/year
• He does not use all of the years (high inflation years when tax parameters were not indexed)

Methods:
• Simple descriptive unconditional exercise
• Uses histograms and kernel density (local smoother of histogram; within a “band” observations further from the central point are weighted less in average)
Results:
• Some evidence of bunching around 1\textsuperscript{st} kink (MTR goes from 0 → 15%)  
  - Figures 2/3  
  - More evidence for single and HH returns  
  - First kink probably the most “visible” to taxpayer. But could the finding be an artifact that those left of 1\textsuperscript{st} kink do not have to file and may not be in data?  
• No evidence at 2\textsuperscript{nd} or later kinks  
• Some evidence of bunching around EITC first kink. Results concentrated for those with self employment income (no effect for those with only wage income)

Implication:
-- Small elasticities  
-- Simulations using extended model again shows no clustering. So these models are not right or elasticities are small or agents do not know where kinks are.  
-- Problematic for research using kinked budget constraint methods
Fig. 2. Distributions around the first kink point, 1988–1997
Fig 3. Dynamics, All taxpayers 1988 versus 1995

- Year 1988 (38,279 obs.)
- Year 1995 (45,038 obs.)
- Kink: 0/15%

Bandwidth h = 2,000

Taxable Income (2000 dollars)
Fig. 7. Density distribution around kink 15 to 28%, 1988–1997
Early Instrumental Variables

Mroz influential study that reviews literature on married women's labor supply.

Identifies instruments that are credible using Hausman specification and overidentification tests.
(Estimate IV with small and large instrument set and test for equality of estimates; can be low power)

Credible instruments: unemp rate, parent's ed, wife's age and ed
Not credible: labor market experience, age hourly earnings, previous reported wages

This study contributed to emerging view that policy variation (taxes) was necessary to identify parameters.

Blundell et al, *Econometrica*, Use demographics, tax reform
Tax Reforms and Labor Supply “New Public Finance”
- Using tax reforms as a “natural experiment” to evaluate the effect of taxes on labor supply (and other outcomes). Can get around problem of endogenous net of tax wages and wages more generally
- Advantage of tax reform: policies can affect some groups and not others, creating natural treatment and control groups.
- We have seen lots of changes in tax laws to provide experiments to examine.

TRA86 Tax Reform Act of 1986
- The program where the most work has been done. Why? (See Auerbach and Slemrod JEL)
- Most significant policy change in postwar period
- Goals of TRA86: Horiz Equity, Efficiency (eliminate tax preferences), Simplicity
- Result: Broaden base + reduce rates MTR
- 1986: 14 brackets, 11% - 50%
- 1990: 5 brackets 0, 15, 28, 33, 28
- Increase standard deduction and personal exemptions
- We will see later papers using this variation to look at impact of taxes on low end (EITC) and high end
Eissa TRA86 and Married Women’s Labor Supply (NBER WP)

Never published (not sure why), but great teaching paper and also very influential paper
-- Established convincingly that married women are sensitive to taxes, have higher elasticity of labor supply
-- Added to our knowledge that participation margin is more sensitive than hours margin
-- Good example of difference in difference methodology
-- most commonly cited DD approach to taxes and labor supply
-- Eissa focuses on high income women because they had the highest reductions in MTR (see figure from paper)
Figure 1

Income Tax Schedule, 1985

Income Tax Schedule, 1989

marginal tax rate

24,700  59,700  124,000
Taxable Income (1985$)
Economics: Secondary Earner Labor supply model
-- Most common approach is to model labor supply of husband and wife sequentially

(1) Husband (or primary earner) maximizes utility ignoring wife (just like single agent model)

\[ \text{Max}(l_h, Y) \ s.t. \ w_h h_h + N = Y \quad \Rightarrow \quad h_h^* \]

(2) Wife (or secondary earner) maximizes utility conditioning on husband’s optimal labor supply decision

Therefore, she takes \( N + w_h h_h^* \) as given
\[ \text{Max}(l_w, C) \ s.t. \ w_w h_w + (w_h h_h^* + N) = C \]
Graph this
Comparative statics of secondary earner model

- Earnings of husband increase $\uparrow$ (through increase in h or w) $\rightarrow$ nonlabor income of wife $\uparrow$ $\rightarrow$ income effect $\rightarrow$ hours and employment of wife $\downarrow$.

- Taxes? Decrease in taxes leads to:
  - $\uparrow$ net nonlabor income $\rightarrow$ hours and employment of wife fall
  - $\uparrow w_w$ $\rightarrow$ hours ($?$), employment $\uparrow$

- KEY: with progressive taxes, she gets the change in MTR which is exogenous to her own labor supply, but comes through her husband. Her first hour MTR is his last hour MTR.
**Empirical Approach**

**Difference in difference (hours and part)**

<table>
<thead>
<tr>
<th>Before</th>
<th>After</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treated</td>
<td>$H_{t0}$</td>
</tr>
<tr>
<td>Control</td>
<td>$H_{co}$</td>
</tr>
</tbody>
</table>

**Treatment** women in $\geq 99^{th}$ percentile of $N + \text{whh}_h$ distribution

**Control** $75^{th}$ percentile or $90^{th}$ percentile

Tradeoff: 90th better control but gets some treatment

$H_{t1} - H_{t0}$ not good if secular trends or other contemporaneous shocks to labor market (secular trends $\uparrow$ over period)

$\left( H_{t1} - H_{t0} \right) - \left( H_{c1} - H_{c0} \right)$ D in D

Nets out $\Delta$ in control group. $\sim$ change that treated would have if they weren’t treated.

*Key* identifying assumption – controls are good comparison

No contemporaneous shock to treated

No assortative mating on unobservables
Data CPS
1984 – 1986 before (83 – 85)
1990 – 1992 after (89 – 91) TRA86 phased in by 88

Predictions?
-- Employment of women in 99th p will rise relative to women in 90th p
\[\rightarrow\] Her MTR ↓ → net wage ↑ → LFP ↑
-- But we have to believe that her net of tax nonlabor income did not change much. Why?
\[\rightarrow\] husband’s MTR ↓ (\[\rightarrow\] ↑ earnings) (or no change if elas small)
\[\rightarrow\] But TRA86 broadened base
\[\rightarrow\] overall effect on her after tax non-labor income is small

* To the extent which his ↓ net earnings are not captured, then, this estimate is an underestimate of total effect.
Figure II

M arginal tax rate

24700 50700 124000
Taxable income (1965)
## Results
### Unconditional difference in difference

<table>
<thead>
<tr>
<th></th>
<th>Ave Y</th>
<th>ΔMTR (Tab IIa)</th>
<th>ΔLFP</th>
<th>D in D</th>
</tr>
</thead>
<tbody>
<tr>
<td>99th p &gt; 90K</td>
<td>-13.9 pp</td>
<td>+9.0 pp</td>
<td></td>
<td></td>
</tr>
<tr>
<td>90th p 67K</td>
<td>-6.9 pp</td>
<td>+4.5 pp</td>
<td>+4.5 pp (2.8) 13%</td>
<td></td>
</tr>
<tr>
<td>75th p 47K</td>
<td>-4.1 pp</td>
<td>+5.3 pp</td>
<td>+3.7 pp (2.8) 12%</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Ave Y</th>
<th>ΔMTR (Tab IIa)</th>
<th>Δhours</th>
<th>D in D</th>
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</thead>
<tbody>
<tr>
<td>99th p &gt; 90K</td>
<td>-13.9 pp</td>
<td>+163</td>
<td></td>
<td></td>
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<tr>
<td>90th p 67K</td>
<td>-6.9 pp</td>
<td>+96</td>
<td>+67 (64.8) 6%</td>
<td></td>
</tr>
<tr>
<td>75th p 47K</td>
<td>-4.1 pp</td>
<td>+55</td>
<td>+108 (65.1) 9%</td>
<td></td>
</tr>
</tbody>
</table>
Conditional D-D

$$\Pr(Work) = \alpha_0 + \alpha_1 Z_{it} + \alpha_2 \text{high}_i + \alpha_3 \text{Post86}_t + \alpha_4 (\text{High}_i * \text{Post86})_{it}$$

$Z_{it} =$ age, educ, # kids, young kids, race, CC, year & state fixed effects

Expectations

$\alpha_2 < 0$ baseline inc. effect

$\alpha_3 > 0$ secular trend

$\alpha_4 > 0$ Main test of TRA86
Results
-- Significant increase in LFP, less for hours
-- Consistent w/ lit showing greater responsiveness on participation margin than hours margin (Mroz, Hausman)
### Table V
Probit Regression Results
Labor Force Participation

<table>
<thead>
<tr>
<th>Variables</th>
<th>Control: 75th Percentile</th>
<th>Control: 90th Percentile</th>
</tr>
</thead>
<tbody>
<tr>
<td>High*Post86</td>
<td>.070 (.072)</td>
<td>.110 (.072)</td>
</tr>
<tr>
<td></td>
<td>.100 (.075)</td>
<td>.120 (.075)</td>
</tr>
<tr>
<td>Post86</td>
<td>.014 (.050)</td>
<td>.064 (.05)</td>
</tr>
<tr>
<td></td>
<td>-.006 (.052)</td>
<td>.038 (.051)</td>
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<tr>
<td>High</td>
<td>-.578 (.050)</td>
<td>-.374 (.050)</td>
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<td></td>
<td>-.746 (.058)</td>
<td>-.411 (.053)</td>
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<tr>
<td>Age</td>
<td>-- .041 (.012)</td>
<td>-- .025 (.014)</td>
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<tr>
<td>Age²</td>
<td>-- -.001 (.0001)</td>
<td>-- -.0006 (.0001)</td>
</tr>
<tr>
<td>Education</td>
<td>-- .005 (.079)</td>
<td>-- -.335 (.110)</td>
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<tr>
<td>Education²</td>
<td>-- .018 (.007)</td>
<td>-- .044 (.009)</td>
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<tr>
<td>Children &lt; 6</td>
<td>-- -.422 (.030)</td>
<td>-- -.387 (.030)</td>
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<td>Black</td>
<td>-- .373 (.094)</td>
<td>-- .407 (.121)</td>
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<tr>
<td>Husband Self-Employed</td>
<td>-- .131 (.042)</td>
<td>-- .037 (.033)</td>
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<tr>
<td>Log Likelihood</td>
<td>-5432 -4935</td>
<td>-5830 -5349</td>
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<tr>
<td>Observations</td>
<td>8886 8886</td>
<td>8817 8817</td>
</tr>
</tbody>
</table>

Regressions in columns (1) and (3) include year dummies. Regressions in columns (2) and (4) include year dummies, state dummies, MSA and central city dummies. Data are March CPS 1984-86 and 1990-92. Standard errors are in parentheses.
Table VIIa  
Predicted Participation Rate  
Probit Estimates

<table>
<thead>
<tr>
<th>Group</th>
<th>Before TRA86</th>
<th>After TRA86</th>
<th>Change</th>
<th>Differences-in-Differences Estimate</th>
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</thead>
<tbody>
<tr>
<td></td>
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<tr>
<td>Control: 75th Percentile</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>High</td>
<td>.460</td>
<td>.497</td>
<td>.037</td>
<td>(8.1%)</td>
</tr>
<tr>
<td>75th Percentile</td>
<td>.741</td>
<td>.739</td>
<td>-.002</td>
<td>(-0.3%)</td>
</tr>
<tr>
<td></td>
<td></td>
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<td></td>
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<tr>
<td>Control: 90th Percentile</td>
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<td></td>
</tr>
<tr>
<td>High</td>
<td>.480</td>
<td>.542</td>
<td>.062</td>
<td>(13.1%)</td>
</tr>
<tr>
<td>90th Percentile</td>
<td>.640</td>
<td>.654</td>
<td>.014</td>
<td>(2.2%)</td>
</tr>
</tbody>
</table>

Predicted Participation is calculated as

\[ P(\bar{y}f) = \Phi (\bar{z} \& \alpha) \]

where \(\bar{z}\) corresponds to the average characteristics in the sample.

Table VIIb  
Elasticity of Participation  
High-Income Group

<table>
<thead>
<tr>
<th>Control Group</th>
<th>Elasticity</th>
</tr>
</thead>
<tbody>
<tr>
<td>75th Percentile</td>
<td>0.4</td>
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<tr>
<td>90th Percentile</td>
<td>0.6</td>
</tr>
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</table>
Criticism of Eissa’s research design [Heckman “Comment”]

General criticisms:
- Heckman does not support structural estimation, but feels that a theoretical DD models are misguided:
  - do not identify any parameters of interest (e.g. elasticities)
  - require strong assumptions (no differential trends across groups)
  - throws away information (in this case using after tax wages in model)

Specific criticisms:
- Assortative mating on unobservables. Trend toward "power couples." Used to be that prof men had nonworking spouses; now more common to have working prof spouse. Yet in middle class more stable situation with working middle class spouse.
- Demand or supply shock to 99th p (e.g. work in different sectors) → different trends for T and C reflecting inequality literature
- Other Δ taxes affect 99th p more than 75th p
- Does tax reform affect selection into “group”? (T, C groups)
- Things to examine in DD model that were not known then:
  - Placebo treatment (use data for pre periods, redo DD using placebo treatment, say comparing year 0 and year -1)
  - Useful (necessary) to plot outcome variables in T and C year on year for whole period; examine whether the trends are similar in pre period. Look for change.
Summary: What do we know about the size of elasticities across groups:

1. Prime Aged Males
   Wage elasticity ~0 mostly > 0 $\tau \uparrow \rightarrow w \downarrow \rightarrow h \downarrow$
   Inc elasticity < 0 but small
   DWL fairly small max 20%/ tax rev.
2. Married women
   Much more sensitive
   Wage elasticity ~0.5 – 1.0
   Inc. elasticity ~ -0.25
   Larger DWL but lower LFPR so this holds it down
   ($\Delta \tau \rightarrow \text{no } \Delta \text{ in } h \text{ for lots of non workers}$)
3. FHH
   In the middle
Econometrics of Kinked Budget Constraints: Convex Budget Set

(Hausman’s model)

Preferences:
\[ h^* = h (w, y, \varepsilon) \]
- \( h \) = observed hours
- \( \varepsilon \) = taste shifter
Comments:
- Virtual income $y_3, y_5$ are a function of observed non-labor income and tax system.
- Need one preference assumption (either labor supply equation, IUF or DUF).
- Assume gross wage is exogenous.
- Assume gross nonlabor income is exogenous.

Ex: Functional form Hausman used for labor supply equation was
\[
hi* = \alpha wi + \beta yi + z \gamma + \epsilon
\]
which implied the following for the IUF:
\[
v(wi, yi) = \exp(\beta wi)[yi + \frac{\alpha}{\beta} wi - \frac{\alpha}{\beta^2} + \frac{z \gamma}{\beta}]
\]
4 Steps in constructing the likelihood function:

1. What do you observe?
2. Identify possible states
3. Determine economic decision rule that justifies each choice
4. Derive probabilities associated with each choice

(Step 1) What do you observe?
Hours (0, or continuous hours worked)
Hourly wage rate, for workers
Nonlabor income
Covariates

(Step 2) States of the World

<table>
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<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
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<td>0</td>
<td>0</td>
<td>h2</td>
<td>h2</td>
<td>h4</td>
<td>h4</td>
<td>h6</td>
</tr>
<tr>
<td></td>
<td>0  &lt; h &lt; h2</td>
<td>h2</td>
<td>h2 &lt; h &lt; h4</td>
<td>h4</td>
<td>h4 &lt; h &lt; h6</td>
<td>h6</td>
<td></td>
</tr>
</tbody>
</table>
Define the labor supply function for each segment:

\[ h(w_i,y_i,\varepsilon) \]  linear labor supply curve for net wage \( w_i \), and net nonlabor income \( y_i \)

for \( i=1,2,3 \)

Ex:

- \( w_1 = w \) (no taxes)
- \( w_3 = w(1-t_1) \) 1\(^{st}\) marginal tax rate
- \( w_5 = w(1-t_2) \) 2\(^{nd}\) marginal tax rate

\( y_1 = N \) observed nonlabor income
\( y_3 \) virtual income \( y_3 \)
\( y_5 \) virtual income \( y_5 \)
(Step 3) Economic Decision Rules

State 0  \( h = 0 \)
\( h( w_1, y_1, \varepsilon ) \leq 0 \)
Desired hours given \( w_1, y_1 \) are <0

State 1  \( 0 < h < h_2 \)
\( h = h( w_1, y_1, \varepsilon ) \)
Desired hours given \( w_1, y_1 \) are between 0 and \( h_2 \)

State 2  \( h = h_2 \)
\( h( w_1, y_1, \varepsilon ) \geq h_2 \quad \text{AND} \quad h( w_3, y_3, \varepsilon ) \leq h_2 \)
Note that being on kink has higher probability than any given point on segment.

State 3  \( h_2 < h < h_4 \)
\( h = h( w_3, y_3, \varepsilon ) \)

State 4  \( h = h_4 \)
\( h( w_3, y_3, \varepsilon ) \geq h_4 \quad \text{AND} \quad h( w_5, y_5, \varepsilon ) \leq h_4 \)

State 5  \( h_4 < h < h_6 \)
\( h = h( w_5, y_5, \varepsilon ) \)

State 6  \( h = h_6 \)
\( h( w_5, y_5, \varepsilon ) \geq h_6 \)
Then translate desired hours rule into rule about unobservable (\( \varepsilon \))
Derive probability that choice was made

(Step 4) Create Likelihood Function

\[
L(h) = \Pi_{i=0,2,4,6} \left[ \Pr(\delta_i = 0) \right]^{\delta_i} \Pi_{i=1,3,5} \left[ f(h | w_i, y_i) \right]^{\delta_i}
\]

where \(\delta_i = 1\) if state i is observed, and = 0 otherwise.